

## Quantum mechanics and reality

Bryce S. DeWitt

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# Quantum mechanics and reality

Could the solution to the dilemma of indeterminism be a universe in which all possible outcomes of an experiment actually occur?

Bryce S. DeWitt

Despite its enormous practical success, quantum theory is so contrary to intuition that, even after 45 years, the experts themselves still do not all agree what to make of it. The area of disagreement centers primarily around the problem of describing observations. Formally, the result of a measurement is a superposition of vectors, each representing the quantity being observed as having one of its possible values. The question that has to be answered is how this superposition can be reconciled with the fact that in practice we only observe one value. How is the measuring instrument prodded into making up its mind which value it has observed?

Of the three main proposals for solving this dilemma, I shall focus on one that pictures the universe as continually splitting into a multiplicity of mutually unobservable but equally real worlds, in each one of which a measurement does give a definite result. Although this proposal leads to a bizarre world view, it may be the most satisfying answer yet advanced.

## Quantum theory of measurement

In its simplest form the quantum theory of measurement considers a world composed of just two dynamical entities, a *system* and an *apparatus*. Both are subject to quantum-mechanical

Bryce DeWitt is professor of physics at the University of North Carolina.

laws, and hence one may form a combined state vector that can be expanded in terms of an orthonormal set of basis vectors

$$|s, A\rangle = |s\rangle|A\rangle \quad (1)$$

where  $s$  is an eigenvalue of some system observable and  $A$  is an eigenvalue of some apparatus observable. (Additional labels have been suppressed for simplicity.) The Cartesian product structure of equation 1 reflects an implicit assumption that, under appropriate conditions, such as the absence of coupling, the system and apparatus can act as if they are isolated, independent and distinguishable. It is also convenient to assume that the eigenvalue  $s$  ranges over a discrete set while the eigenvalue  $A$  ranges over a continuum.

Suppose that the state of the world at some initial instant is represented by a normalized vector of the form

$$|\Psi_0\rangle = |\psi\rangle|\Phi\rangle \quad (2)$$

where  $|\psi\rangle$  refers to the system and  $|\Phi\rangle$  to the apparatus. In such a state the system and apparatus are said to be "uncorrelated." For the apparatus to learn something about the system the two must be coupled together for a certain period, so that their combined state will not retain the form of equation 2 as time passes. The final result of the coupling will be described by the action of a certain unitary operator  $U$

$$|\Psi_1\rangle = U|\Psi_0\rangle \quad (3)$$

Because the apparatus observes the system and not vice versa, we must choose a coupling operator  $U$  that reflects this separation of function. Let  $U$  have the following action on the basis vectors defined in equation 1 (or on some similar basis):

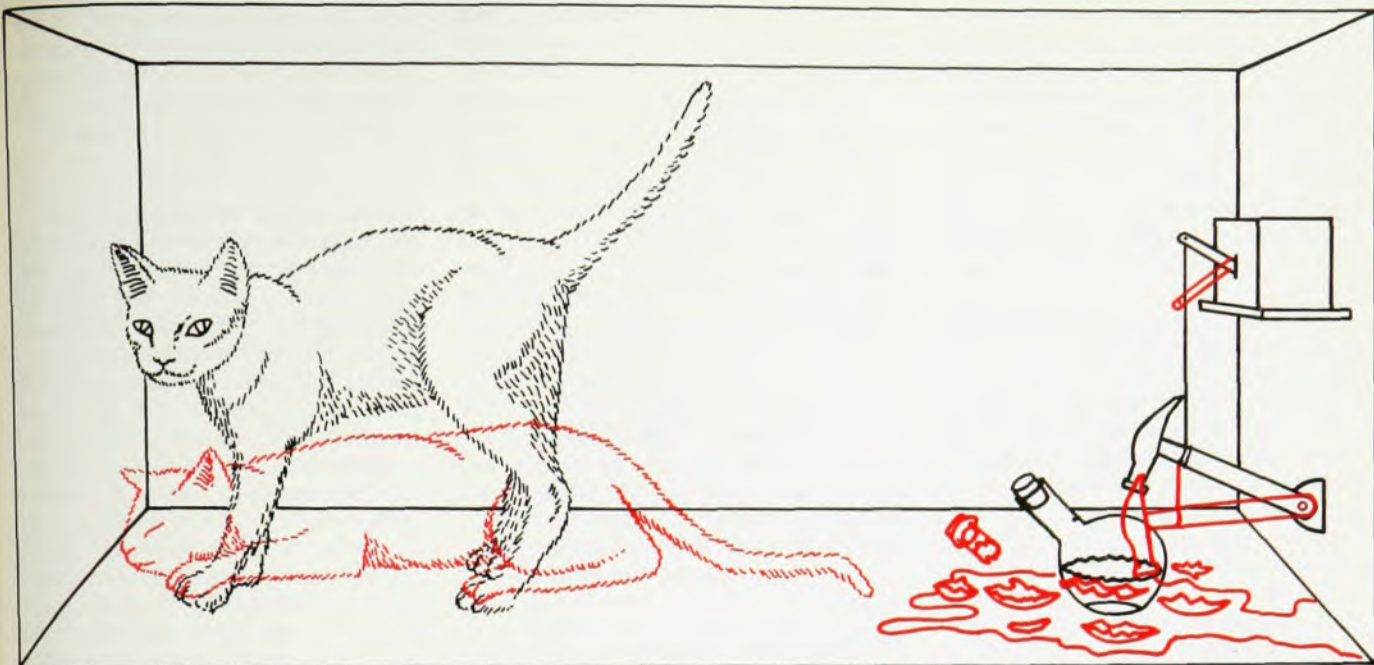
$$U|s, A\rangle = |s, A + gs\rangle = |s\rangle|A + gs\rangle \quad (4)$$

Here,  $g$  is a coupling constant, which may be assumed to be adjustable. If the initial state of the system were  $|s\rangle$  and that of the apparatus were  $|A\rangle$  then this coupling would be said to result in an "observation," by the apparatus, that the system observable has the value  $s$ . This observation or "measurement," would be regarded as "stored" in the apparatus "memory" by virtue of the permanent shift from  $|A\rangle$  to  $|A + gs\rangle$  in the apparatus state vector.

## Is this definition adequate?

This particular choice for  $U$ , essentially formulated by John von Neumann,<sup>1</sup> is frequently criticized because it is not sufficiently general and because it artificially delimits the concept of measurement. Some writers<sup>2</sup> have also insisted that the process described by equation 4 merely prepares the system and that the measurement is not complete until a more complicated piece of apparatus observes the outcome of the preparation.





**Schrödinger's cat.** The animal trapped in a room together with a Geiger counter and a hammer, which, upon discharge of the counter, smashes a flask of prussic acid. The counter contains a trace of radioactive material—just enough that in one hour there is a 50% chance one of the nuclei will decay and therefore an equal chance the cat will be poisoned. At the end of the hour the total wave function for the system will have a form in which the living cat and the dead cat are mixed in equal portions. Schrödinger felt that the wave mechanics that led to this paradox presented an unacceptable description of reality. However, Everett, Wheeler and Graham's interpretation of quantum mechanics pictures the cats as inhabiting two simultaneous, noninteracting, but equally real worlds.

It is perfectly true that laboratory measurements are much more complicated than that described by equation 4 and often involve interactions that do not establish precise correlations between pairs of observables such as  $s$  and  $A$ . However, apart from such noncorrelative interactions, every laboratory measurement consists of one or more sequences of interactions, each essentially of the von Neumann type. Although it is only the results of the final interactions with the recording devices that we usually regard as being stored, each von Neumann-type "apparatus" in every sequence leading to a final interaction may itself be said to possess a memory, at least momentarily. This memory differs in no fundamental way from that of the sophisticated automaton (apparatus-plus-memory sequence) at the end of the line. It is the elementary component that must be understood if we are to understand quantum mechanics itself.

In his original analysis of the measurement process,<sup>1</sup> von Neumann assumed that the coupling between system and apparatus leaves the system observable  $s$  undisturbed. Most of his conclusions would have remained unaffected had he removed this restriction, and we are not making such an assumption here. Although measurements of the nondisturbing type do exist, more frequently the observable suffers a change. It can

nevertheless be shown<sup>3</sup> that if suitable devices are used, such as the compensation devices introduced by Niels Bohr and Leon Rosenfeld in their analysis of electromagnetic-field measurements,<sup>4</sup> the apparatus can record what the value of the system observable would have been without the coupling. For this reason, we work in a modified version of the so-called "interaction picture," in which only that part of the state vector that refers to the apparatus changes during the coupling interval.

If the coupling is known, the hypothetical undisturbed system observable may be expressed in terms of the actual dynamical variables of system *plus* apparatus. Hence, the operator of which this observable is an eigenvalue is not itself hypothetical, and no inconsistency will arise if we take it to be the right-hand side of equation 4.

#### Infinite regression

Consider now what happens to the initial state vector in equation 2 as a result of the measurement process of equation 4. Using the orthonormality and assumed completeness of the basis vectors, we easily find that

$$|\Psi_1\rangle = \sum_s c_s |s\rangle |\Phi[s]\rangle \quad (5)$$

where

$$c_s = \langle s | \Psi \rangle \quad (6)$$

$$|\Phi[s]\rangle = \int [A + gs] \Phi(A) dA \quad (7)$$

$$\Phi(A) = \langle A | \Phi \rangle \quad (8)$$

The final state vector in equation 5 does not represent the system observable as having any unique value—unless, of course,  $|\psi\rangle$  happens to be one of the basis vectors  $|s\rangle$ . Rather it is a linear superposition of vectors  $|s\rangle |\Phi[s]\rangle$ , each of which represents the system observable as having assumed one of its possible values and the apparatus as having observed that value. For each possibility the observation will be a good one, that is, capable of distinguishing adjacent values of  $s$ , provided

$$\Delta A \ll g \Delta s \quad (9)$$

where  $\Delta s$  is the spacing between adjacent values and  $\Delta A$  is the variance in  $A$  about its mean value relative to the distribution function  $|\Phi(A)|^2$ . Under these conditions we have

$$\langle \Phi[s] | \Phi[s'] \rangle = \delta_{ss'} \quad (10)$$

In other words, the wave function of the apparatus takes the form of a packet that is initially single but subsequently splits, as a result of the coupling to the system, into a multitude of mutually orthogonal packets, one for each value of  $s$ .

Here the controversies over the interpretation of quantum mechanics start. For most people, a state like that of equation 5 does not represent the actual



occurrence of an observation. They conceive the apparatus to have entered a kind of schizophrenic state in which it is unable to decide what value it has found for the system observable. At the same time they can not deny that the coupling chosen between system and apparatus would, in the classical theory, have led to a definite outcome. They therefore face a crisis. How can they prod the apparatus into making up its mind?

The usual suggestion is to introduce a second apparatus to get at the facts simply by looking at the first apparatus to see what it has recorded. But an analysis carried out along the above lines quickly shows that the second apparatus performs no better than the first. It too goes into a state of schizophrenia. The same thing happens with a third apparatus, and a fourth, and so on. This chain, known as "von Neumann's catastrophe of infinite regression," only makes the crisis worse.

### Change the rules

There are essentially three distinct ways of getting out of the crisis. The first is to change the rules of the game by changing the theory, the object being to break von Neumann's infinite chain. Eugene Wigner is the most distinguished proponent of this method. Taking a remarkably anthropocentric stand, he proposes that the entry of the measurement signal into the consciousness of an observer is what triggers the decision and breaks the chain.<sup>5</sup> Certainly the chain had better be broken at this point, as the human brain is usually where laboratory-measurement sequences terminate. One is reminded of the sign that used to stand on President Truman's desk: "The buck stops here."

Wigner does not indulge in mere handwaving; he actually sketches a possible mathematical description of the conversion from a pure to a mixed state, which might come about as a result of the grossly nonlinear departures from the normal Schrödinger equation that he believes must occur when conscious

beings enter the picture. He also proposes that a search be made for unusual effects of consciousness acting on matter.<sup>5</sup>

Another proponent of the change-the-rules method is David Bohm.<sup>6,7</sup> Unlike Wigner, who does not wish to change the theory below the level of consciousness, Bohm and his school want to change the foundations so that even the first apparatus is cured of its schizophrenia. This they do by introducing so-called "hidden variables." Whatever else may be said of hidden-variable theories, it must be admitted that they do what they are supposed to. The first such theory<sup>6</sup> in fact worked too well; there was no way of distinguishing it experimentally from conventional quantum mechanics. More recent hidden-variable theories are susceptible to possible experimental verification (or disproof).<sup>7</sup>

### The Copenhagen collapse

The second method of escaping the von Neumann catastrophe is to accept the so-called "conventional," or "Copenhagen," interpretation of quantum mechanics. (Reference 8 contains a selected list of papers on this topic.) In speaking of the adherents of this interpretation it is important to distinguish the active adherents from the rest, and to realize that even most textbook authors are not included among the former. If a poll were conducted among physicists, the majority would profess membership in the conventionalist camp, just as most Americans would claim to believe in the Bill of Rights, whether they had ever read it or not. The great difficulty in dealing with the activists in this camp is that they too change the rules of the game but, unlike Wigner and Bohm, pretend that they don't.

According to the Copenhagen interpretation of quantum mechanics, whenever a state vector attains a form like that in equation 5 it immediately collapses. The wave function, instead of consisting of a multitude of packets, reduces to a single packet, and the vector

$|\Psi_1\rangle$  reduces to a corresponding element  $|s\rangle\Phi[s]$  of the superposition. To which element of the superposition it reduces one can not say. One instead assigns a probability distribution to the possible outcomes, with weights given by

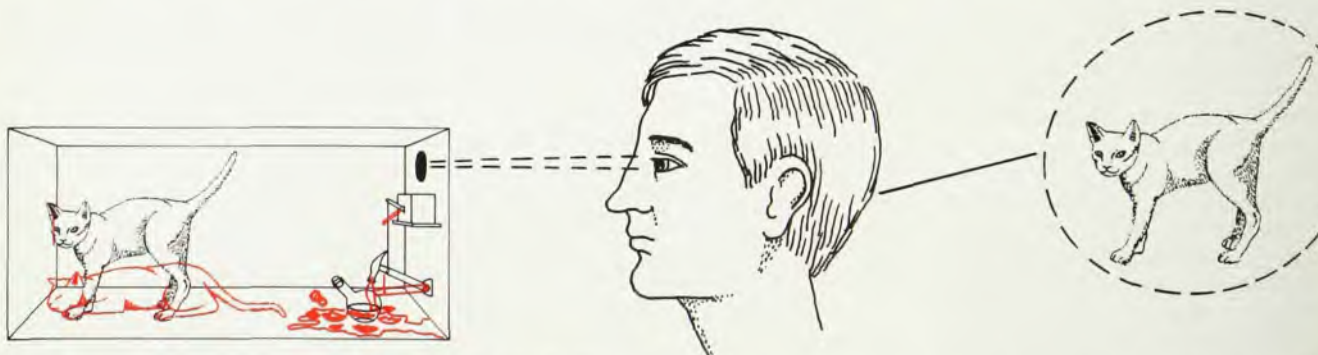
$$w_s = |c_s|^2 \quad (11)$$

The collapse of the state vector and the assignment of statistical weights do not follow from the Schrödinger equation, which generates the operator  $U$  (equation 4). They are consequences of an external *a priori* metaphysics, which is allowed to intervene at this point and suspend the Schrödinger equation, or rather replace the boundary conditions on its solution by those of the collapsed state vector. Bohm and Wigner try to construct explicit mechanisms for bringing about the collapse, but the conventionalists claim that it does not matter how the state vector is collapsed. To them the state vector does not represent reality but only an algorithm for making statistical predictions. In fact, if the measurement involves a von Neumann chain they are even willing to leave the state vector uncollapsed over an arbitrary number of links, just so long as it is treated as collapsed somewhere along the line.

The Copenhagen view promotes the impression that the collapse of the state vector, and even the state vector itself, is all in the mind. If this impression is correct, then what becomes of reality? How can one treat so cavalierly the objective world that obviously exists all around us? Einstein, who opposed to his death the metaphysical solution of the Copenhagen school, must surely have expressed himself thus in his moments of private indignation over the quantum theory. I am convinced that these sentiments also underlie much of the current dissatisfaction with the conventional interpretation of quantum mechanics.

### Historical interpretations

This problem of the physical interpretation of the quantum theory haunted its earliest designers. In 1925 and



"The buck stops here." Wigner's solution to the dilemma of the schizophrenic apparatus is to claim that the entry of the measurement signal into the consciousness of a human observer triggers the decision as to which of the possible outcomes is observed—that is, whether the cat is alive or dead.



1926 Werner Heisenberg had just succeeded in breaking the quantum theory from its moorings to the old quantum rules. Through the work of Max Born, Pascual Jordan, Erwin Schrödinger, P. A. M. Dirac and Heisenberg himself, this theory soon acquired a fully developed mathematical formalism. The challenge then arose of elucidating the physical interpretation of this formalism independently of anything that had gone on before.

Heisenberg attempted to meet this challenge by inventing numerous thought experiments, each of which was subjected to the question: "Can it be described by the formalism?" He conjectured that the set of experiments for which the answer is "yes" is identical to the set permitted by nature." To put the question in its most extreme form in each case meant describing the complete experiment, including the measuring apparatus itself, in quantum-mechanical terms.

At this point Bohr entered the picture and deflected Heisenberg somewhat from his original program. Bohr convinced Heisenberg and most other physicists that quantum mechanics has no meaning in the absence of a classical realm capable of unambiguously recording the results of observations. The mixture of metaphysics with physics, which this notion entailed, led to the almost universal belief that the chief issues of interpretation are epistemological rather than ontological: The quantum realm must be viewed as a kind of ghostly world whose symbols, such as the wave function, represent potentiality rather than reality.

### The EWG metatheorem

What if we forgot all metaphysical ideas and started over again at the point where Heisenberg found himself in 1925? Of course we can not forget everything; we will inevitably use 45 years of hindsight in attempting to restructure our interpretation of quantum mechanics. Let us nevertheless try

- ▶ to take the mathematical formalism of quantum mechanics as it stands without adding anything to it
- ▶ to deny the existence of a separate classical realm
- ▶ to assert that the state vector never collapses.

In other words, what if we assert that the formalism is all, that nothing else is needed? Can we get away with it? The answer is that we can. The proof of this assertion was first given in 1957 by Hugh Everett<sup>10</sup> with the encouragement of John Wheeler<sup>11</sup> and has been subsequently elaborated by R. Neill Graham.<sup>12</sup> It constitutes the third way of getting out of the crisis posed by the catastrophe of infinite regression.

Everett, Wheeler and Graham (EWG) postulate that the real world,

or any isolated part of it one may wish for the moment to regard as *the* world, is faithfully represented solely by the following mathematical objects: a vector in a Hilbert space; a set of dynamical equations (derived from a variational principle) for a set of operators that act on the Hilbert space, and a set of commutation relations for the operators (derived from the Poisson brackets of the classical theory by the quantization rule, where classical analogs exist). Only one additional postulate is then needed to give physical meaning to the mathematics. This is the postulate of complexity: The world must be sufficiently complicated that it be decomposable into systems and apparatuses.

Without drawing on any external metaphysics or mathematics other than the standard rules of logic, EWG are able, from these postulates, to prove the following metatheorem: *The mathematical formalism of the quantum theory is capable of yielding its own interpretation.* To prove this metatheorem, EWG must answer two questions:

- ▶ How can the conventional probability interpretation of quantum mechanics emerge from the formalism itself?
- ▶ How can any correspondence with reality be achieved if the state vector never collapses?

### Absolute chance

Before giving the answers to these questions, let us note that the conventional interpretation of quantum mechanics confuses two concepts that really ought to be kept distinct—probability as it relates to quantum mechanics and probability as it is understood in statistical mechanics. Quantum mechanics is a theory that attempts to describe in mathematical language a world in which chance is not a measure of our ignorance but is absolute. It should inevitably lead to states, like that of equation 5, that undergo multiple fission, corresponding to the many possible outcomes of a given measurement. Such behavior is built into the formalism. However, precisely because quantum-mechanical chance is *not* a measure of our ignorance, we ought not to tamper with the state vector merely because we acquire new information as a result of a measurement.

The obstacle to taking such a lofty view of things, of course, is that it forces us to believe in the reality of all the simultaneous worlds represented in the superposition described by equation 5, in each of which the measurement has yielded a different outcome. Nevertheless, this is precisely what EWG would have us believe. According to them the real universe is faithfully represented by a state vector similar to

that in equation 5 but of vastly greater complexity. This universe is constantly splitting into a stupendous number of branches, all resulting from the measurementlike interactions between its myriads of components. Moreover, every quantum transition taking place on every star, in every galaxy, in every remote corner of the universe is splitting our local world on earth into myriads of copies of itself.

### A splitting universe

I still recall vividly the shock I experienced on first encountering this multiworld concept. The idea of  $10^{100}+$  slightly imperfect copies of oneself all constantly splitting into further copies, which ultimately become unrecognizable, is not easy to reconcile with common sense. Here is schizophrenia with a vengeance. How pale in comparison is the mental state of the imaginary friend, described by Wigner,<sup>5</sup> who is hanging in suspended animation between only two possible outcomes of a quantum measurement. Here we must surely protest. None of us feels like Wigner's friend. We do not split in two, let alone into  $10^{100}+$ ! To this EWG reply: To the extent that we can be regarded simply as automata and hence on a par with ordinary measuring apparatuses, the laws of quantum mechanics do not allow us to feel the splits.

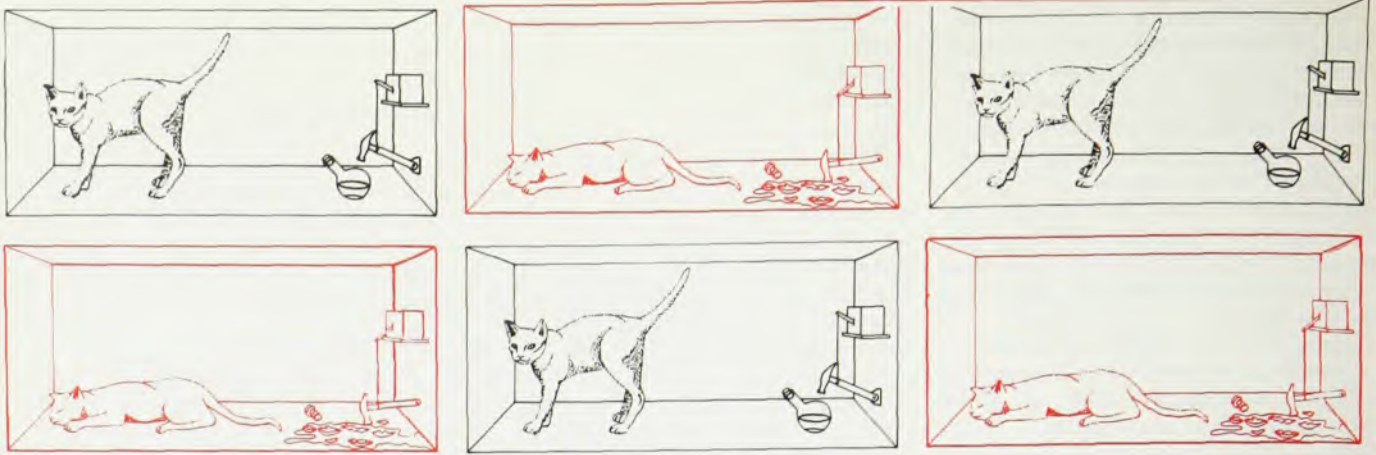
A good way to prove this assertion is to begin by asking what would happen, in the case of the measurement described earlier by equations 4 and 5, if one introduced a second apparatus that not only looks at the memory bank of the first apparatus but also carries out an independent direct check on the value of the system observable. If the splitting of the universe is to be unobservable the results had better agree.

The couplings necessary to accomplish the desired measurements are readily set up. The final result is as follows (see reference 13): The state vector at the end of the coupling interval again takes the form of a linear superposition of vectors, each of which represents the system observable as having assumed one of its possible values. Although the value varies from one element of the superposition to another, not only do both apparatuses within a given element observe the value appropriate to that element, but also, by straightforward communication, they agree that the results of their observations are identical. The splitting into branches is thus unobserved.

### Probability interpretation

We must still discuss the questions of the coefficients  $c_s$  in equations 6 and 7. EWG give no *a priori* interpretation to these coefficients. In order to find





**The Copenhagen collapse.** This interpretation pictures the total wave function as collapsing to one state of the superposition and assigns a probability that the wave function will collapse to a given state. Only for repetition on an ensemble of cats would live and dead cats be equally real.

an interpretation they introduce an apparatus that makes repeated measurements on an ensemble of identical systems in identical states. The initial state then has the form

$$|\Psi_0\rangle = |\psi_1\rangle|\psi_2\rangle \dots |\Phi\rangle \quad (12)$$

where

$$\langle s|\psi_i\rangle = c_s \quad \text{for all } i \quad (13)$$

and the successive measurements are described in terms of basis vectors

$$|s_1\rangle|s_2\rangle \dots |A_1, A_2, \dots\rangle \quad (14)$$

If the apparatus observes each system exactly once, in sequence, then the  $n$ th measurement is represented by a unitary transition of the form

$$U_n(|s_1\rangle|s_2\rangle \dots |A_1, A_2, \dots, A_n, \dots\rangle = |s_1\rangle|s_2\rangle \dots |A_1, A_2, \dots, A_n + g_{s_n}, \dots\rangle \quad (15)$$

After  $N$  measurements the state vector in equation 12 is changed to

$$|\Psi_N\rangle = \sum_{s_1, s_2, \dots} c_{s_1} c_{s_2} \dots |s_1\rangle|s_2\rangle \dots |\Phi[s_1, s_2, \dots, s_n]\rangle \quad (16)$$

where

$$|\Phi[s_1, s_2, \dots]\rangle = \int dA_1 \int dA_2 \dots |A_1 + g_{s_1, A_1} + g_{s_2, A_2} + \dots\rangle \Phi(A_1, A_2, \dots) \quad (17)$$

$$\Phi(A_1, A_2, \dots) = \langle A_1, A_2, \dots | \Phi \rangle \quad (18)$$

Although every system is initially in exactly the same state as every other, the apparatus does not generally record a sequence of identical values for the system observable, even within a single element of the superposition of equation 16. Each memory sequence  $s_1, s_2, \dots, s_N$  yields a certain distribution of possible values for the system observable, and each distribution may be subjected to a statistical analysis. The first and simplest part of such an analysis is the calculation of the relative frequency function of the distribution:

$$f(s; s_1 \dots s_N) = \frac{1}{N} \sum_{n=1}^N \delta_{s, s_n} \quad (19)$$

Let us introduce the function

$$\delta(s_1 \dots s_N) = \sum_s [f(s; s_1 \dots s_N) - w_s]^2 \quad (20)$$

where the  $w$ 's are any positive numbers that add up to unity. This is the first of a hierarchy of functions that measure the degree to which the sequence  $s_1 \dots s_N$  deviates from a random sequence with weights  $w_s$ . Let us choose for the  $w$ 's the numbers defined in equation 11, and let us introduce an arbitrarily small positive number  $\epsilon$ . We shall call the sequence  $s_1 \dots s_N$  "first random" if  $\delta(s_1 \dots s_N) < \epsilon$  and "non-first-random" otherwise.

Suppose now we remove from the superposition of equation 16 all those elements for which the apparatus memory sequence is non-first-random. Denote the result by  $|\Psi_N^\epsilon\rangle$ . This vector has the remarkable property that it differs negligibly from  $|\Psi_N\rangle$  in the limit  $N \rightarrow \infty$ . More precisely,

$$\lim_{N \rightarrow \infty} (|\Psi_N\rangle - |\Psi_N^\epsilon\rangle) = 0 \quad \text{for all } \epsilon > 0 \quad (21)$$

A proof will be found in reference 13.

A similar result is obtained if  $|\Psi_N^\epsilon\rangle$  is redefined by excluding, in addition, elements of the superposition whose memory sequences fail to meet any finite combination of the infinity of other requirements for a random sequence. The conventional probability interpretation of quantum mechanics thus emerges from the formalism itself. Nonrandom memory sequences in equation 16 are of measure zero in the Hilbert space, in the limit as  $N$  goes to infinity. Each automaton in the superposition sees the world obeying the familiar statistical quantum laws. However, there exists no outside agency that can designate which branch of the

superposition is to be regarded as the real world. All are equally real, and yet each is unaware of the others. These conclusions obviously admit of immediate extension to the world of cosmology. Its state vector is like a tree with an enormous number of branches. Each branch corresponds to a possible universe-as-we-actually-see-it.

### Maverick worlds

The alert reader may now object that the above argument is circular, that in order to derive the *physical* probability interpretation of quantum mechanics, based on sequences of observations, we have introduced a *nonphysical* probability concept, namely that of the measure of a subspace in Hilbert space. This concept is alien to experimental physics because it involves many elements of the superposition at once, and hence many simultaneous worlds, that are supposed to be unaware of one another.

The problem that this objection raises is like many that have arisen in the long history of probability theory. Actually, EWG do not in the end exclude any element of the superposition. All the worlds are there, even those in which everything goes wrong and all the statistical laws break down. The situation is no different from that which we face in ordinary statistical mechanics. If the initial conditions were right, the universe-as-we-see-it could be a place in which heat sometimes flows from cold bodies to hot. We can perhaps argue that in those branches in which the universe makes a habit of misbehaving in this way, life fails to evolve; so no intelligent automata are around to be amazed by it.

It is also possible that maverick worlds are simply absent from the grand superposition. This could be the case if ordinary three-space is compact and the universe is finite. The wave func-



tion of a finite universe must itself contain only a finite number of branches. It simply may not have enough fine structure to accommodate maverick worlds. The extreme smallness of the portion of Hilbert space that such worlds would have to occupy becomes obvious when one compares the length of a Poincaré cycle, for even a small portion of the universe, to a typical cosmological time scale.

### Questions of practicality

The concept of a universal wave function leads to important questions regarding the practical application of the quantum-mechanical formalism. If I am part of the universe, how does it happen that I am able, without running into inconsistencies, to include as much or as little as I like of the real world of cosmology in my state vector? Why should I be so fortunate as to be able, in practice, to avoid dealing with the state vector of the universe?

The answer to these questions is to be found in the statistical implications of sequences of measurements of the kind that led us to the state vector of equation 16. Consider one of the memory sequences in this state vector. This memory sequence defines an average value for the system observable, given by

$$\langle s \rangle_{s_1, \dots, s_N} = \sum_s f(s; s_1, \dots, s_N) \quad (22)$$

If the sequence is random, as it is increasingly likely to be when  $N$  becomes large, this average will differ only by an amount of order  $\epsilon$  from the average

$$\langle s \rangle = \sum_s s w_s \quad (23)$$

But the latter average may also be expressed in the form

$$\langle s \rangle = \langle \psi | s | \psi \rangle \quad (24)$$

where  $|\psi\rangle$  is the initial state vector of any one of the identical systems and  $s$  is the operator of which the  $s$ 's are the eigenvalues. In this form the basis vectors  $|s\rangle$  do not appear. Had we chosen to introduce a different apparatus, designed to measure some observable  $r$  not equal to  $s$ , a sequence of repeated measurements would have yielded in this case an average approximately equal to

$$\langle r \rangle = \langle \psi | r | \psi \rangle \quad (25)$$

In terms of the basis vectors  $|s\rangle$  this average is given by

$$\langle r \rangle = \sum_{s, s'} c_s^* \langle s | r | s' \rangle c_{s'} \quad (26)$$

Now suppose that we first measure  $s$  and then perform a statistical analysis on  $r$ . We introduce a second apparatus that performs a sequence of observations on a set of identical two-component systems all in identical states given by

the vector  $|\Psi_1\rangle$  of equation 5. Each of the latter systems is composed of one of the original systems together with an apparatus that has just measured the observable  $s$ . In view of the packet orthogonality relations, given by equation 10, we shall find for the average of  $r$  in this case

$$\langle r \rangle = \langle \Psi_1 | r | \Psi_1 \rangle = \sum_s w_s \langle s | r | s \rangle \quad (27)$$

The averages in equations 26 and 27 are generally not equal. In equation 27, the measurement of  $s$ , which the first apparatus has performed, has destroyed the quantum interference effects that are still present in equation 26. Thus the elements of the superposition in equation 5 may be treated as if they were members of a statistical ensemble.

This result is what allows us, in practice, to collapse the state vector after a measurement has occurred, and to use the techniques of ordinary statistical mechanics, in which we change the boundary conditions upon receipt of new information. It is also what permits us to introduce systems having well defined initial states, without at the same time introducing the apparatuses that prepared the systems in those states. In brief, it is what allows us to start at any point in any branch of the universal state vector without worrying about previous or simultaneous branches.

We may, in principle, restore the interference effects of equation 26 by bringing the apparatus packets back together again. But then the correlations between system and apparatus are destroyed, the apparatus memory is wiped out and no measurement results. If one attempts to maintain the correlations by sneaking in a second apparatus to "have a look" before the packets are brought back together, then the state vector of the second apparatus must be introduced, and the separation of its packets will destroy the interference effects.

### Final assessment

Clearly the EWG view of quantum mechanics leads to experimental predictions identical with those of the Copenhagen view. This, of course, is its major weakness. Like the original Bohm theory<sup>6</sup> it can never receive operational support in the laboratory. No experiment can reveal the existence of the "other worlds" in a superposition like that in equations 5 and 6. However, the EWG theory does have the pedagogical merit of bringing most of the fundamental issues of measurement theory clearly into the foreground, and hence of providing a useful framework for discussion.

Moreover a decision between the two

interpretations may ultimately be made on grounds other than direct laboratory experimentation. For example, in the very early moments of the universe, during the cosmological "Big Bang," the universal wave function may have possessed an overall coherence as yet unimpaired by condensation into non-interfering branches. Such initial coherence may have testable implications for cosmology.

Finally, the EWG interpretation of quantum mechanics has an important contribution to make to the philosophy of science. By showing that formalism alone is sufficient to generate interpretation, it has breathed new life into the old idea of a direct correspondence between formalism and reality. The reality implied here is admittedly bizarre. To anyone who is awestruck by the vastness of the presently known universe, the view from where Everett, Wheeler and Graham sit is truly impressive. Yet it is a completely causal view, which even Einstein might have accepted. At any rate, it has a better claim than most to be the natural end product of the interpretation program begun by Heisenberg in 1925.

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