The Quantum Hourglass

Approaching time measurement with Quantum Information Theory

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Abstract

In this thesis a general non-relativistic operational model for quantum clocks called quantum hourglass is devised. Taking results from the field of Quantum Information Theory the limitations of such time measurement devices are explored in different physical and information-theoretic scenarios. It will be shown for certain cases that the time resolution of the quantum hourglass is limited by its power consumption.

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1 The Past

Time enters Quantum Theory in manifold ways. Since the early days of quantum theory this led to heated discussions about its nature. In a letter Heisenberg wrote to Pauli in November 1925, he commented:

> Your problem of the time sequence plays, of course, a fundamental role, and I had figured out for my own private use (Hausgebrauch) something about it. First I believe that one can distinguish between "a coarse and a finer" time sequence. If a point in space does not assume in the new theory a definite role or can be formulated only symbolically, then the same is true for an instant of time of an event. But there always will exist a coarse time sequence, like a coarse position in space - that is, within our geometric visualization one will be able to carry out a coarse description. I think it might be possible that this coarse description is perhaps the only thing that can be demanded from the formalism [of quantum mechanics].[2]

In the following year Pauli wonders if the absence of a clear definition of the times of transition in Einstein's probabilistic treatment of absorption and emission, is due to a fundamental cause or the incompleteness of the theory. He concludes that, "this is very much debated, yet still an unsolved issue." [3, 4, 5]

In a footnote in the second edition of the "Handbuch der Physik" seven years later he gave a formal argument why the idea of introducing a self-adjoint time operator as the canonical conjugate to the Hamiltonian should

be fundamentally abandoned. The introduction of such an operator would unavoidably lead to a new energy eigenstate with an unbounded spectrum. This is very undesirable as most physical systems of interest have bounded, semi-bounded or discrete spectra. This argument often referred to as a theorem, however enforced the view that time is a simple parameter, governing the evolution of quantum as well as classical systems. In recent years the development of research introducing such a time operator with relaxed conditions, e.g. not requiring self-adjointness, has led to a better understanding of times of arrival [4, 6].

Our inability to formulate a consistent theory with an observable for time brings another problem along, namely, the question 'how well can we observe time'. This is really important as we don't have access to the parameter t directly. One of the first widely recognized attempts to answer this question was given by Mandelstam and Tamm in 1945 [7]. In their observable-dependent energy-time uncertainty relation they refer to time characteristics of average expectation values. This approach however has its limitations [4, 5]. In this thesis a possible answer to the former question will be given more in the spirit of above quote by Heisenberg.

For this purpose a general non-relativistic operational model for quantum clocks called quantum hourglass will be introduced that has the ability to distinguish 'coarser and finer' time sequences depending on its implementation. Taking results from the field of Quantum Information Theory and other fundamental physical theories the limitations and properties of such time measurement devices are explored in different physical and information-theoretic scenarios. Moreover it will be shown that for cases where the interaction between the two parts (see section 3) of the quantum hourglass can be modeled by a qubit channel the time resolution of the quantum hourglass is limited by its power consumption.

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The thesis will be organized as follows: In section 2 the basic concepts and mathematical notations will be introduced. Section 3 will include the introduction and the definition of the quantum hourglass. Furthermore different physical and information-theoretic scenarios will be discussed. Section 4 is a short excursus into the realms of synchronization and causality. Section 5 will discuss open questions and section 6 contains the conclusion.

2 The Context

In this work we make use of quantum systems A,B with corresponding finite-dimensional Hilbert spaces $\mathcal{H}_A, \mathcal{H}_B$ where $|A|, |B|$ denote their dimensionality. According to the postulates of quantum mechanics the Hilbert space of a composite system is given by the tensor product $\mathcal{H}_{AB} := \mathcal{H}_A \otimes \mathcal{H}_B$. Furthermore the space of homomorphisms $\mathcal{M} : \mathcal{H}_A \to \mathcal{H}_B$ is written as $\mathrm{Hom}(\mathcal{H}_A, \mathcal{H}_B)$. The state of a quantum system is again according to the postulates of quantum mechanics represented by a ket $|\phi\rangle_A$ element of some Hilbert space $\mathcal{H}_A$, hence exploiting the isomorphism $\mathcal{H}_A \cong \mathrm{Hom}(\mathbb{C}, \mathcal{H}_A)$. Moreover, $\mathrm{End}(\mathcal{H})$ is the set of endomorphisms on $\mathcal{H}$, i.e. the homomorphisms from a Hilbert space $\mathcal{H}$ to itself.

The eigenvalues of some operator $O_A \in \mathrm{End}(\mathcal{H})$ are denoted as $\lambda_i(O_A)$ and its singular values as $s_i(O_A)$, where the subscript specifies the system the operator acts on. For the hermitian adjoint of some operator O we use the expression $O^\dagger$, additionally the set of hermitian operators on $\mathcal{H}$ is defined by $\mathrm{Herm}(\mathcal{H}) := \{H \in \mathrm{End}(\mathcal{H}) : H = H^\dagger\}$. The trace of an Operator is defined as

$$\mathrm{Tr}(O_A) \equiv \sum_i \langle e_i | O_A | e_i \rangle \tag{1}$$

where $\{e_i\}_i$ is any orthonormal basis of $\mathcal{H}_A$.

The identity operator $\mathbb{1} \in \text{End}(\mathcal{H})$ maps any vector $|\phi\rangle \in \mathcal{H}$ to itself. It can be written as

$$\mathbb{1} = \sum_i |e_i\rangle\langle e_i| \tag{2}$$

for any orthonormal basis $\{e_i\}_i$ of $\mathcal{H}$.

The set of positive semi-definite operators that act on $\mathcal{H}$ is denoted as $\mathcal{P}(\mathcal{H})$ and is defined as

$$\mathcal{P}(\mathcal{H}) := \{O \in \text{End}(\mathcal{H}) : \lambda_i(O) \geq 0 \quad \forall\, i\}. \tag{3}$$

In the following $\mathcal{S}_{\leq}(\mathcal{H}) := \{\rho \in \mathcal{P}(\mathcal{H}) : 0 < \text{Tr}(\rho) \leq 1\}$ will be the set of subnormalized quantum states acting on $\mathcal{H}$ and $\mathcal{S}_{=}(\mathcal{H}) := \{\rho \in \mathcal{P}(\mathcal{H}) : \text{Tr}(\rho) = 1\}$ be the set of all normalized states. We will also make use of the trace norm which is defined as

$$||O||_1 := \text{Tr}\sqrt{O^\dagger O}. \tag{4}$$

To quantify the distance between any two quantum states $\rho, \sigma \in \mathcal{S}_{\leq}$ we use the generalized fidelity, defined as

$$F(\rho, \sigma) := \left|\left|\sqrt{\rho}\sqrt{\sigma}\right|\right|_1 - \sqrt{(1 - \text{Tr}\rho)(1 - \text{Tr}\sigma)} \tag{5}$$

which in the case that either ρ or σ is normalized the above expression reduces to the standard fidelity

$$F(\rho, \sigma) := \left|\left|\sqrt{\rho}\sqrt{\sigma}\right|\right|_1. \tag{6}$$

A linear map $\mathcal{E} \in \text{Hom}(\mathcal{H}_A, \mathcal{H}_B)$ is said to be positive if

$$\mathcal{E}(\rho) \geq 0 \quad \forall \quad \rho \in \mathcal{S} \tag{7}$$

and it is called completely positive if the map $\mathcal{E} \otimes \mathbb{1}_C$ is positive for any

Hilbert space $\mathcal{H}_C$. Furthermore if $\mathrm{Tr}\left(\mathcal{E}(\rho)\right) = \mathrm{Tr}\left(\rho\right)$ holds for all $\rho \in \mathcal{S}$ the map $\mathcal{E}$ is called trace preserving and trace-nonincreasing if $\mathrm{Tr}\left(\mathcal{E}(\rho)\right) \geq \mathrm{Tr}\left(\rho\right)$.

In the following the reduced Planck 'Wirkungsquantum'

$$\hbar = \frac{h}{2\pi} = 1.054571726(47) \times 10^{-34} \mathrm{J\ s}, \tag{8}$$

the speed of light in vacuum

$$c = 299792458\frac{\mathrm{m}}{\mathrm{s}} \tag{9}$$

and the Stefan-Boltzmann constant

$$k_B = 5.670373(21) \times 10^{-8}\frac{\mathrm{W}}{\mathrm{m^2K^4}}, \tag{10}$$

based on CODATA 2010 [9].

2.1 Margolus-Levitin bound

Until Margulos and Levitin published their classic paper in 1998, bounds placed on the minimal time needed for a state to evolve into an orthogonal state were always thought of in terms of the standard deviation of energy ΔE [10], i.e.

$$t_\perp \geq \frac{h}{4\Delta E}. \tag{11}$$

But these bounds don't place a limit on the velocity with which systems with bounded energy can evolve, since ΔE can be made arbitrarily large. Bounds based directly on the average energy of a system were also proposed [11, 12] but were only applicable to the rate of communication in bits, which can also lead to complications [13]. In the following the results by Margulos and Levitin will be discussed in a more detailed manner.

Let $|\psi_0\rangle$ be an arbitrary quantum state. It can be written as a superposition of energy eigenstates, i.e.

$$|\psi_0\rangle = \sum_n c_n |E_n\rangle, \tag{12}$$

where we restrict ourselves to systems with a discrete spectrum. Furthermore assume w.l.o.g that the energy eigenstates are ordered in a non-decreasing fashion. Also we choose $E_0 = 0$.

Define the time it takes for $|\psi_0\rangle$ to evolve into an orthogonal state as $\tau_\perp$. Then Margolus and Levitin showed that for a fixed average energy E, that

$$t_\perp \geq \frac{h}{4E}. \tag{13}$$

To prove this statement it is helpful to start with the observation that $|\psi_0\rangle$, after evolving for some time t, can be written as

$$|\psi_t\rangle = \sum_n c_n e^{-i\frac{E_n t}{\hbar}} |E_n\rangle. \tag{14}$$

Now one can define the inner product of the initial state and the evolved state as a function of t. Let

$$S(t) := \langle \psi_0 | \psi_t \rangle = \sum_{n=0}^{\infty} |c_n|^2 e^{-i\frac{E_n t}{\hbar}} \tag{15}$$

and look at the smallest possible t s.t. $S(t) = 0$. In order to achieve this task

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let us look at the real part of S

$$\Re(S) = \sum_{n=0}^{\infty} |c_n|^2 \cos \frac{E_n t}{\hbar} \tag{16}$$

$$\geq \sum_{n=0}^{\infty} |c_n|^2 \left(1 - \frac{2}{\pi} \left(\frac{E_n t}{\hbar} + \sin \frac{E_n t}{\hbar} \right) \right) \tag{17}$$

$$= 1 - \frac{2E}{\pi\hbar} t + \frac{2}{\pi} \Im(S) \tag{18}$$

where in the second line the inequality $\cos x \geq 1 - \frac{2}{\pi}(x + \sin x)$ for $x \geq 0$ was used. As $S = 0$ if and only if $\Im(S) = 0$ and $\Re(S) = 0$, from the last line the condition

$$1 - \frac{4E}{h} t \leq 0 \tag{19}$$

is obtained. Thus equation (13) has been established.

2.1.1   A generalized bound

The inequality used to attain equation (17) may not be optimal in all cases. In 2006 Kosinski and Zych gave an elementary proof for bounds on the speed of quantum evolution [14] that gave a unified way to derive the Mandelstam-Tamm [7] bound as well as the Margolus-Levitin one [10]. A few months afterwards Zych together with Zielinkski used this technique to generalize the Margolus-Levitin bound [15]. It is quite insightful to have a closer look. By means of the spectral theorem one writes

$$\langle \psi | e^{\frac{-iHt}{\hbar}} | \psi \rangle = \int e^{\frac{-iEt}{\hbar}} d\langle \psi | P_E | \psi \rangle$$

$$= \int \cos \left(\frac{Ht_\perp}{\hbar} \right) d\langle \psi | P_E | \psi \rangle - i \int \sin \left(\frac{Ht_\perp}{\hbar} \right) d\langle \psi | P_E | \psi \rangle$$

where the spectral measure in the decomposition of H was denoted P_E, $H = \int E dP_E$. For orthogonal states i.e. states that evolved for some $t_\perp$ it follows that

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$$\langle\psi|\cos\left(\frac{Ht_\perp}{\hbar}\right)|\psi\rangle = 0 = \langle\psi|\sin\left(\frac{Ht_\perp}{\hbar}\right)|\psi\rangle \tag{20}$$

If one now assumes that an inequality of the form

$$f(x) \geq A\sin(x) + B\cos(x) \tag{21}$$

to hold for $x \geq 0$ (for the derivation of the Mandelstam-Tamm relation one needs to assume such an inequality to hold for all x) together with the observation that the expectation value of non-negative functions is also non-negative we have that

$$\langle\psi|f\left(\frac{Ht}{\hbar}\right) - A\sin\left(\frac{Ht}{\hbar}\right) - B\cos\left(\frac{Ht}{\hbar}\right)|\psi\rangle = \tag{22}$$

$$= \int\left[f\left(\frac{Et}{\hbar}\right) - A\sin\left(\frac{Et}{\hbar}\right) - B\cos\left(\frac{Et}{\hbar}\right)\right]d\langle\psi|P_E|\psi\rangle \geq 0 \tag{23}$$

W.l.o.g. the ground state energy was set to zero here (as in the previous section). Now the combination of equation (20) and (23) already places restrictions on $t_\perp$, namely

$$\langle\psi|f\left(\frac{Ht_\perp}{\hbar}\right)|\psi\rangle \geq 0. \tag{24}$$

In order to derive a different bound than the one given by Margolus and Levitin, Zielinski and Zych continue by using the following inequality

$$x^\alpha - \frac{\pi^\alpha}{2} + \frac{\pi^\alpha}{2}\cos x + \alpha\pi^{\alpha-1}\sin x \geq 0 \tag{25}$$

which holds for all $x \geq 0$ and $\alpha > 0$. This inequality reduces to the inequality used to derive equation (17) if one sets $\alpha = 1$. For all $\alpha > 0$ it now holds that

$$t_\perp \geq \frac{\pi\hbar}{2^{\frac{1}{\alpha}} \langle\psi|E^\alpha|\psi\rangle^{\frac{1}{\alpha}}} \geq 0 \tag{26}$$

provided that $|\psi\rangle$ belongs to the domain of H^α. For certain states choosing $\alpha \neq 1$ can lead to a much better bound [15].

Note that these results can also be extended for the case of mixed states through Uhlmann's theorem [16]. Let be $\rho \in \mathcal{S}$ and $|\xi\rangle$ some purification of ρ. For a trivial evolution of the purifying system i.e. the total system evolves with $H \otimes \mathbb{1}$, it holds that

$$F(\rho, \rho(t)) \geq |\langle\xi|\xi(t)\rangle|^2. \tag{27}$$

2.1.2 The adiabatic case

Let us shortly review what happens in the case of an adiabatic evolution. Consider a system where a time-dependent Hamiltonian $H(t)$ governs the evolution of the given quantum system through the Schrödinger equation

$$i\hbar\frac{d}{dt}|\psi(t)\rangle = H(t)|\psi(t)\rangle. \tag{28}$$

Adiabaticity means that $H(t)$ is slowly varying. This very case has been analyzed by the authors of [17]. In the following a little review of their results will be given.

Let $E_0(t)$ and $E_1(t)$ be the energy of the ground and the first excited state for some time dependent Hamiltonian $H(t)$.

Then the minimum gap between these two quantities is given by

$$\omega_{min} = \min_{0 \leq t \leq T} \left[E_0(t) - E_1(t)\right]. \tag{29}$$

Furthermore denote the maximum value of the matrix element $\frac{dH(t)}{dt}$ between the eigenstates by

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$$\Omega_{max} = \max_{0 \le t \le T} \left| \left\langle \frac{dH(t)}{dt} \right\rangle_{1,0} \right| = \max_{0 \le t \le T} \left| \langle E_1(t)| \frac{dH(t)}{dt}|E_0(t)\rangle \right|. \qquad (30)$$

Now the adiabatic theorem [18] tells us that if we first prepare our system in the ground state at $t = 0$ and then let the system evolve according to $H(t)$ for some time T then

$$|\langle E_0(T)|\psi(T)\rangle|^2 \ge 1 - \epsilon^2 \qquad (31)$$

given that

$$\Omega_{max}\omega_{min}^{-2} \le \epsilon \qquad (32)$$

with $0 \le \epsilon \le 1$.

Such systems have recently caught a lot of attention since it was established that adiabatic quantum computing is possible [19]. This is very favourable since it is a nice way around the problem of energy relaxation. Here the focus shall lie on the Margolus-Levitin like bound given for systems evolving adiabatically. For such systems Andrecut and Ali proved in [17] by non-linear interpolation that the minimum time from one state to an orthogonal state is given by

$$T(E) = \frac{\pi\hbar}{2E\epsilon} \qquad (33)$$

for a constant minimum energy gap $\omega_{min} = E$.

2.2 Communication rate vs power consumption

A very nice application of the Margolus-Levitin bound was given by Lloyd in 2003 [20]. Here his results are reviewed in more detail.

Let us start with the simplest model of a quantum channel namely the qubit

channel. That is a channel that transmits a qubit from Alice to Bob [21]. Alice and Bob are both in posession of two-level quantum system. Alice holds an initial state $|\psi\rangle$ while Bob's system is initialized in some standard state, e.g. $|0\rangle$. After the transmission Bob has received the state $|\psi\rangle$. By choosing $|\psi\rangle$ one can either transmit classical information, 0 or 1[1], or quantum information $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ with $|\alpha|^2 + |\beta|^2 = 1$.

Now if Alice wants to transmit a 1 to Bob, Bob's state has be rotated by an angle of π. As we have seen in the previous sections such a transformation in time $t_\perp$ requires an energy of the complete system above the ground state of $E_1 \geq \frac{\pi\hbar}{2t_\perp}$. For an average energy $\langle E \rangle = p_0 E_0 + p_1 E_1$ with p_0 and p_1 denoting the probabilities that a 0 or a 1 is sent respectively. For a channel where a 1 is sent with probability $\frac{1}{2}$ Alice transmits a bit to Bob at a rate

$$C_1 = \frac{1}{t_\perp} = \sqrt{4\frac{P}{\pi\hbar}}. \tag{34}$$

Then the power is given by

$$P = \frac{\langle E \rangle}{t_\perp}. \tag{35}$$

This limitation of the communication rate by the square root of the power invested is very akin to the fundamental limits of computation derived in [22]. The bound for the rate of transformation holds whether the transformation considered is due to communication, computation or work. This should not be surprising from a standpoint where the laws of nature are concerned how information can be communicated and processed.

Another interesting example considered in [20] is the case where one uses M uncoupled qubit channels. In this case an overall rate is $\sqrt{M}$ times greater than the one that the single qubit channel can attain. If one allows for

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[1]Encoded into the states $|\psi\rangle = |0\rangle$ and $|1\rangle$ respectively.

coupling of these channels, even a rate of [20]

$$C_M = M\sqrt{\frac{2P}{(1 - 2^{-M})\pi\hbar}}.$$ (36)

is possible. Thus communicating M bits in the same time $t_\perp$ with the same overall power P available. Evidently entanglement [23, 24] can help in increasing the number of bits communicated within some time but cannot increase the speed of the transformation.

It should also be noted that the rate C_1 can be enhanced by using strategies that avoid sending a 1 or by using error correction techniques where one avoids rotating for full angle π [20]. The maximum limit of such enhancements is not known and therefore remains an interesting open question to be tackled.

3 Quantum Hourglass

The simple model for a non-relativistic quantum clock which will be called quantum hourglass or simply hourglass consists of two parts. The first part is the system that is used as some sort of cursor and the second will be a register system where information about the first should be stored available to read out.

Since in the quantum world it is not possible to just look at the cursor of a clock as if nothing has happened, such a splitting seems very intuitive. Aside the interaction Hamiltionian H_{int} coupling the cursor system C to the register system R both systems are assumed to have their autonomous evolution governed by their local Hamiltonians, H_C and H_R respectively. In general the evolution of the systems and the mutual interaction between them can be seen as completely positive (trace-nonincreasing) maps due to the Choi–Jamiołkowski isomorphism [26, 27, 28]. Let the initial state of the cursor system C and register system R be given by

$$\rho_C \in \mathcal{P}(\mathcal{H}_C) \quad \text{and} \quad \rho_R \in \mathcal{P}(\mathcal{H}_R) \tag{37}$$

where $d_C := dim\{\mathcal{H}_C\}$, $d_R := dim\{\mathcal{H}_R\}$ are the dimensions of the cursor and the register system. Here it is omitted specifying any more details in order to keep as much generality as possible. This enables the construction of different types of hourglasses by specifying more details rigorously. This price is being paid willingly. Somewhat along the line of a quotation (presumably) by Bohr; 'Truth and clarity are complementary' [29, 30].

To avoid having to make assumptions about some global background time and/or its properties the quantum hourglass is placed in an isolated lab. In the case that the laboratory is truly isolated that the overall state ('going to

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the church of the larger Hilbert space') is pure

$$|\psi\rangle \in \mathcal{H}_{CRL}. \tag{38}$$

Furthermore some local parameter t that is not accessible to the hourglass is assumed. A schematic illustration of the hourglass is given in figure 1. Making further assumptions about the parameter t is not necessary because from the work of Wootters and Page [31, 32] we know that we can replace the notion of time by correlations. Thus even in a stationary pure state we can observe a time evolution in a subsystem from the perspective of a different subsystem. This astonishing and philosophically striking result shows us another peculiarity of quantum theory and has recently found its way to experiment [33].
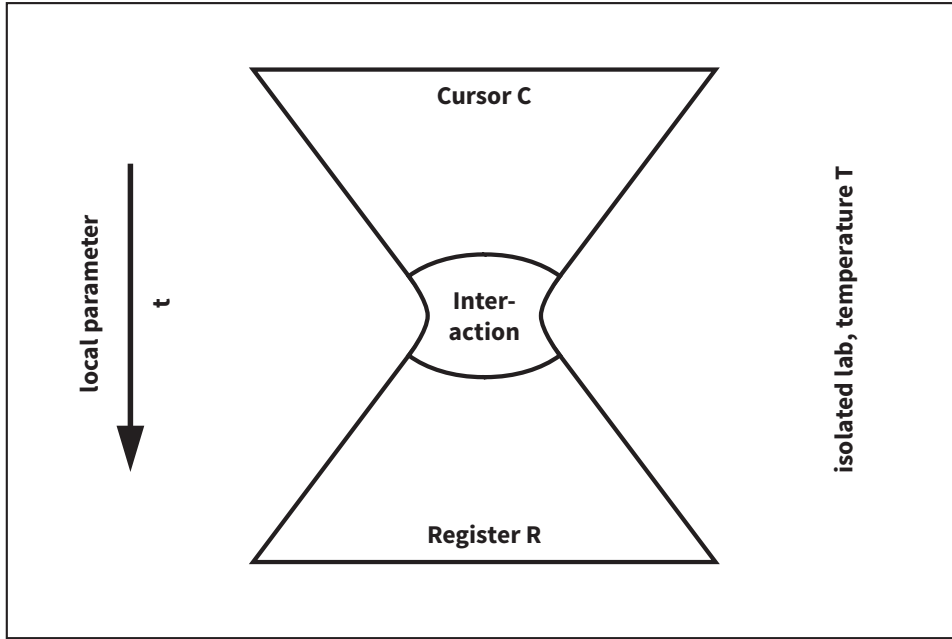

Figure 1: Quantum hourglass in an isolated lab with temperature T

Note that the quantum hourglass described here includes the model used in

the philosophical comment [34] where the author defines an hourglass as an isolated quantum system that can radiate. This assertion is justified as the radiation emitted by the quantum system can be seen as a register system in the sense defined above. Furthermore, parallel to this work a clock construction that is equivalent to a quantum hourglass with trivial evolution on the register system R was developed by Ranković, Liang and Renner [35].
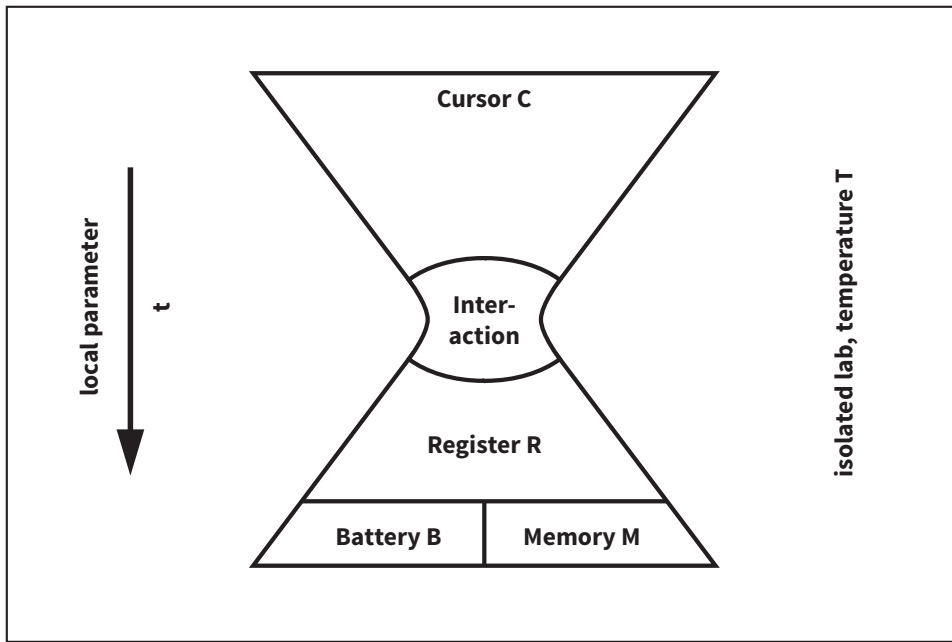

Figure 2: Quantum hourglass with battery B and memory M in an isolated lab with temperature T

In addition to the above mentioned splitting of the quantum hourglass, it can be equipped with two more operationally different parts, namely a battery system B and a memory system M as depicted in figure 2. Especially the battery system B will turn out to be necessary as it will be shown in the next chapter that a time measuring device as the quantum hourglass has a power consumption related to its time resolution in case that the interaction

can be modeled by a qubit channel.

3.1 Time resolution

Assume that H_{int} can be modeled as a completely positive and trace-preserving map $\mathcal{E}$ with which the system C acts on the register R. Thus from a quantum information point of view the quantum hourglass can be seen as a quantum channel from the cursor system C to the register system R. Let us review the simplest case, namely, the qubit channel. Already here one can make a powerful statement about the limitations on the time resolution of such an hourglass. As shown in chapter 2 there is a minimal time that a quantum system needs to undergo an evolution to reach a state orthogonal to the initial state. For a clock to 'tick' something has to happen. As discussed in section 2.2 sending a 0 down the channel does not cause any transformation of the register. Nevertheless sending a 1 implies that the register qubit needs to be rotated by an angle π. This in turn requires an average energy of the overall system above the ground state of at least $E_1 \geq \frac{\pi\hbar}{2t_\perp}$. Thus a 'tick' is defined as sending a 1 in this case and consequently $\frac{1}{t_\perp}$ is the highest possible 'tick' rate, i.e. the maximal time resolution of the hourglass. Because of equation (35) we know that on average [20]

$$P \geq \frac{\pi\hbar}{4t_\perp^2}. \tag{39}$$

This sets a fundamental limit to the time resolution of clocks where the interaction can be modeled as qubit channel.

As mentioned in section 2.2 one can make use of M entangled channels to send M bits with the same amount of power P. This however does not improve the time resolution of the quantum hourglass as one still needs $t_\perp$ to accomplish this task. Still it is surely useful to increase its accuracy and/or precision. Note that the power consumed does not need to be definitely

lost, i.e. dissipated to the environment, but can possibly be recovered if the procedure is implemented reversibly. The power needed to run the hourglass is provided by the battery B.

3.2 The hourglass as a quantum computer

Building on the ideas of Feynman [36] a class of Hamiltonian based quantum computing models have been developed in the last years. Time-independent or slowly-varying Hamiltonians are used for the composition of a sequence of local unitaries, i.e. a quantum circuit. The L gates

$$U = U_L \dots U_2 U_1. \tag{40}$$

are encoded into the history state $|\psi\rangle$ where the second system keeps track of the progress

$$|\psi\rangle = \frac{1}{\sqrt{L+1}} \sum_{t=0}^{L} (U_t \dots U_2 U_1 |\phi\rangle_C) \otimes |t\rangle_R. \tag{41}$$

It is clear that these two systems can be identified with the cursor system C on the one hand and the register system R on the other hand, already indicated in the notation above. Systems of the above form are computationally universal, for example using the 3-local railroad-switch Hamiltonian presented in [37]. Thus the quantum hourglass is as well a universal model of quantum computation if the restrictions placed are weak enough to allow for the implementation of a universal set of gates.

This does not come suprising as one may think of the Turing machine as a clock as well. When the writting head is seen as the dynamical system or cursor and the initial state of the register and the computation to be performed is known, reading out the register gives information on the time passed.

The existing analysis of such systems however focuses on obtaining the result of the computation $|\phi\rangle_t = U |\phi\rangle$ and not using a (possibly trivial) computa-

tion as cursor for a time measurement. Still there are some results obtained in the recent past that are very interesting for this purpose and they are discussed further in light of this new context. Especially using the correspondence of our simple non-relativistic model for a quantum clock with Hamiltonian quantum computing models to measure time algorithmically seems very appealing.

Also note reference [38] where the authors use a model that is an hourglass where the cursor system is taken to perform a continuous time quantum walk (they call their model 'machine') to write down the explicit Lindblad evolution of the system. Furthermore they give a series of very interesting results on speed and entropy of such systems and explore possible ways to reduce the entropy generation in the clocked system.

3.2.1 CNOT hourglass

A possible implementation of an hourglass with time resolution saturating equation (39) can be given in terms of the CNOT hourglass depicted in figure 3.
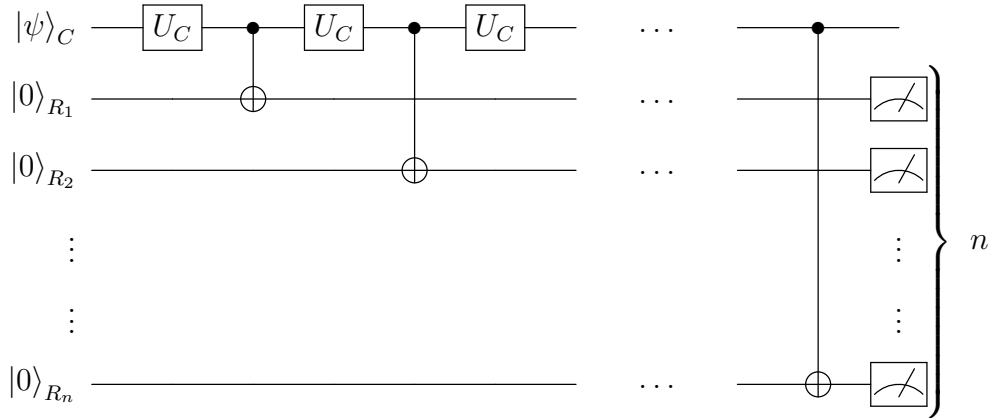

Figure 3: CNOT hourglass with trivial evolution on the ancilla systems

Here the cursor system system C is represented by a qubit, i.e. $d_C = 2$ and the interaction between system C and the register R is given by the CNOT

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gate, which flips the register bit if the cursor system C is found in state $|1\rangle$. Thus this operation can be represented by the following matrix

$$CNOT := \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}. \tag{42}$$

Initializing the n register qubits in the $|0\rangle$ state the state of the hourglass after the first step would be given by

$$|\psi_1\rangle_{CR} = CNOT_{CR_1} \left(U_C \otimes U_{R_1} \otimes \cdots \otimes U_{R_n} \left(|\psi\rangle_C \otimes |0\rangle_{R_1} \otimes \cdots \otimes |0\rangle_{R_n} \right) \right) \tag{43}$$

Assuming that this process saturates the Margolus-Levitin bound given in section 2.1 it can be performed at a rate of $\frac{2E}{\pi\hbar}$ per second. Here E is the average energy of the logic gate that performs the operation.

After running this process $m \leq n$ times the hourglass will be in the state

$$|\psi_m\rangle_{CR} = U_{CR_1\ldots R_m} \left(|\psi\rangle_C \otimes |0\rangle_{R_1} \otimes \cdots \otimes |0\rangle_{R_n} \right) \tag{44}$$

where

$$U_{CR_1\ldots R_m} = \left(CNOT_{CR_m} \left(U_C \otimes U_{R_1} \otimes \cdots \otimes U_{R_n} \right) \right)$$
$$\ldots \left(CNOT_{CR_1} \left(U_C \otimes U_{R_1} \otimes \cdots \otimes U_{R_n} \right) \right).$$

The measured time can be read out from the register system R by measuring the ancilla qubits in system R. In the optimal case one recovers m 'ticks' which gain meaning of time by the scale given through the energy of the interaction, thus $t = mt_\perp = \frac{mh}{2E}$.

3.2.2 SWAP hourglass - quantum logic clock

Recent experimental advances enabled physicists to construct a clock performing metrology at the 17th decimal place [39]. The authors of the mentioned publication named the design quantum logic clock due to its inherent use of quantum logic [40, 41]. The quantum hourglass model is well suited to describe this new kind of clock. In principle it is a more complex form of a CNOT clock where the mutual motional degree of freedom (register system R_1) of two ions in the same trap is used to map the state of the spectroscopy ion (cursor system C) onto the logic ion (register system R_2).

First the system as a whole is cooled to its ground state. Then a coherent pulse U_C is applied near the transition resonance of the spectroscopy ion chosen as a time scale. A π pulse on the red sideband causes it to swap its state with the motional degree of freedom [40]. Finally the state is swapped onto the state of the logic ion by applying a π pulse on its red sideband. The logic ion can then be projectively read out without affecting the state of the spectroscopy too much. The action of U_C on the cursor system can be generally written as

$$U_C \left|0\right\rangle_C = \alpha \left|0\right\rangle_C + \beta \left|1\right\rangle_C \tag{45}$$

where $|\alpha|^2 + |\beta|^2 = 1$, hence

$$\left|\psi\right\rangle_{CR_1R_2} = \left|0\right\rangle_C \otimes \left|0\right\rangle_{R_1} \otimes \left|0\right\rangle_{R_2} \tag{46}$$

$$\longrightarrow^{U_C} (\alpha \left|0\right\rangle_C + \beta \left|1\right\rangle_C) \otimes \left|0\right\rangle_{R_1} \otimes \left|0\right\rangle_{R_2} \tag{47}$$

$$\longrightarrow^{\pi_C} \left|0\right\rangle_C \otimes (\alpha \left|0\right\rangle_{R_1} + \beta \left|1\right\rangle_{R_1}) \otimes \left|0\right\rangle_{R_2} \tag{48}$$

$$\longrightarrow^{\pi_{R_2}} \left|0\right\rangle_C \otimes \left|0\right\rangle_{R_1} \otimes (\alpha \left|0\right\rangle_{R_2} + \beta \left|1\right\rangle_{R_2}). \tag{49}$$

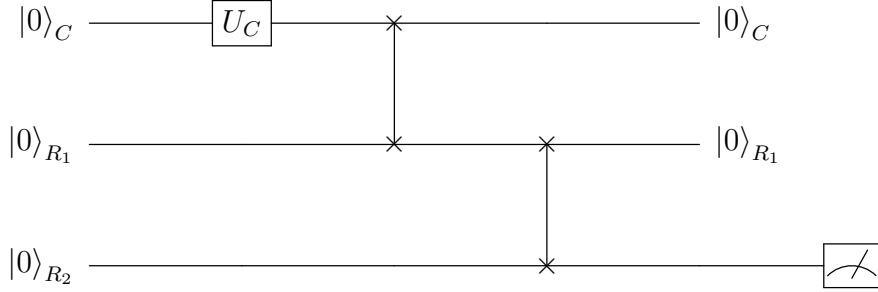The process described is shown below in figure 4.

Figure 4: SWAP hourglass - quantum logic clock in a quantum circuit representation

The SWAP gate above depicted above can be represented by the following matrix

$$
SWAP := \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{50}
$$

By iterating this process one can obtain the values of α and β thereby determining the ion's transition frequency given precise knowledge of the spectroscopy probe frequency.

The reminiscence to the CNOT hourglass is highlighted by the identity in figure 5.
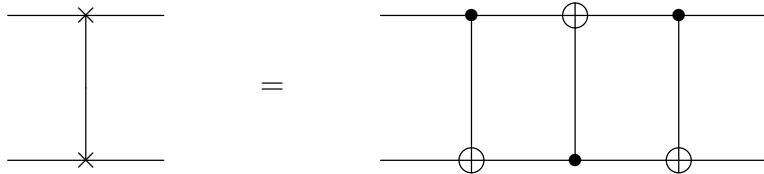

Figure 5: SWAP-CNOT identity [42]

3.2.3 Algorithmic hourglass - Mesoscopic implementation

Since in experiment any observation is bound to be finite one can think of an optimal time measurement as counting the smallest steps of change possible. Thus one can think of the time measurement problem as a counting problem. In quantum information theory algorithms are a well-liked approach to solve problems (approximately) that cannot be solved directly. In 1998 Brassard, Høyer and Tapp introduced the quantum counting algorithm [43] by complementing Grover's iteration [44] with Shor's factoring algorithm [45]. Here these ideas are used for the ability to think about an hourglass that counts algorithmically. Based on the results of Lesovik, Suslov and Blatter [46] a possible implementation on a mesoscopic scale is introduced. Such a device may prove useful in the future where the miniaturization of machines leads to a need of time measuring devices operating in the quantum regime.

Using more technical terms the state of the passing particles $|n\rangle \in \mathcal{H}_C$, where $\mathcal{H}_C$ is the Hilbert space spanned by the number states $|0\rangle, |1\rangle |2\rangle, \ldots, |N\rangle$, is considered. The information encoded in $|n\rangle$ we want to read out using the K qubits in the register system R. The ingenious trick used by Lesovik, Suslov and Blatter [46] is to count the passage of the particles of system C in a Fourier basis in the register. Thus the passage of the particles transforms the initial state of the register qubits $\mathcal{F}(|0\rangle_R)$ in system R into a state

$$\mathcal{F}(|n\rangle_R) \propto \sum_{j=0}^{2^K-1} e^{2i\pi nj} |j\rangle_R \,.$$

This means that such a coupling enables one to count the passage of the particles in a binary fashion $|n\rangle_R = |n_1, n_2, \ldots, n_K\rangle$ where $n = n_1 2^{K-1} + n_2 2^{K-2} + \cdots + n_K 2^0$. Applying a inverse quantum Fourier transformation to the final state one would be able to access the full information encoded in the K register qubits through a single-shot measurement. It is also possible
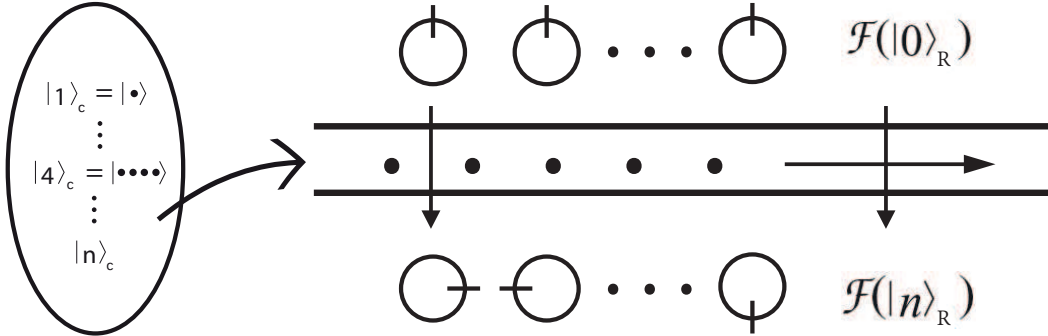
Figure 6: Mesoscopic implementation of an algorithmic hourglass based on [46]

to use a semi-classical scheme where a sequential conditional read out of the register qubits is used [46].

Depending on the system such an hourglass is implemented in the time scales giving physical meaning to the measurement outcome vary. Under dc bias conditions (with voltage V) single-electrons are seperated by the voltage time $t = \frac{h}{eV}$ when their wavefunctions are generated by unit-flux voltage pulses of Lorentzian shape [47, 48]. Here $e = 1.602176565(35) \times 10^{-19} C$ [9] denotes the single-electron charge. An alternative approach works in the quantum Hall regime injecting single-electrons from a quantum dot into an edge channel. Here the typical times $t = \frac{h}{\delta T}$ are found on the nanosecond scale [49]. T is denotes the tunneling probability and δ the level seperation between states in the quantum dot.

3.3 Thermal hourglass

There is an intimate connection between time and temperature. Evidence for such a relationship can already be found in the early universe. As most astrophysical cosmologist see it today there was a precise connection between both [50]. This relation at the time when relativistic particles dominated the

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density of the universe is thought to be

$$t \propto T^{-2} \tag{51}$$

where T from now on denotes the temperature. Another very interesting evidence lies in the connection between the 'time-asymmetry', i.e. the perception that we live in a world that has a preferred direction of time and the second law of thermodynamics has been discussed in great detail over the last decades (see [51, 52, 53, 54, 55, 56, 57] and references therein). From the quantum information theory point view this is very interesting discussion as there exists a fundamental feature within the theory that may well underly this phenomena, the data processing inequality. Loosely speaking it states that the amount of information in a quantum system cannot be increased by acting on it locally. Formally the data processing inequality can be written as

$$\tilde{H}(A|BC)_\rho \leq \tilde{H}(A|B)_\rho \tag{52}$$

where $\tilde{H}$ is some entropy measure and $\tilde{H}(A|B)$ denotes the conditional entropy of some quantum system $\rho_{AB} \in \mathcal{S}$. The first proof was given by Lieb and Ruskai in the early 70's [58]. The data processing inequality comes with a lot of interesting features and has also has a scientifically thrilling history, but this thesis shall not be concerned with telling this story and refers the interested reader to the existing literature [59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70].

As Page and Wootter [31, 32] have shown an overall static (pure) state $|\psi\rangle$ can still inhibit an time evolution if one takes the perspective of one of the subsystems due to the correlation of the overall system. Recently Popescu, Short and Winter [71] came up with a very astonishing result namely that for almost all pure states a subsystem will be thermal. Due to the leakage of information to the environment the entanglement is expected to increase.

This reinforces nearly forgotten ideas brought up by Lloyd in his 1988 PhD thesis [72] where he suggests that the arrow of time is an arrow of increasing correlations.

As we have seen above there is a cluster of problems concerning the notion of time intertwining the different well-established physical theories we know today. It seems that the problem of the conception of time lies in the core of a possible unification of General Relativity, Thermodynamics and Quantum Theory. A quite radical idea solving some of these problems is given by the introduction of thermal time [73].

Here the possible workings of two thermal hourglasses measuring time in a well-controlled thermodynamic environment are sketched. In an interesting arXiv preprint it was shown last month that even a single qubit can be used as a thermometer [74]. Further Stace shows in a paper [75] that a Heisenberg scaling ($\sim \frac{1}{N}$) is attainable in thermometry for thermometers that are not fully thermalized. In order to do so he maps the problem of temperature measurement for a bosonic system to the phase estimation problem. Although such proposal is probably far from being implementable in experiment (if one believes in Moore's law not too far) it shows that for specific systems a clock can be seen as a thermometer and vice versa.
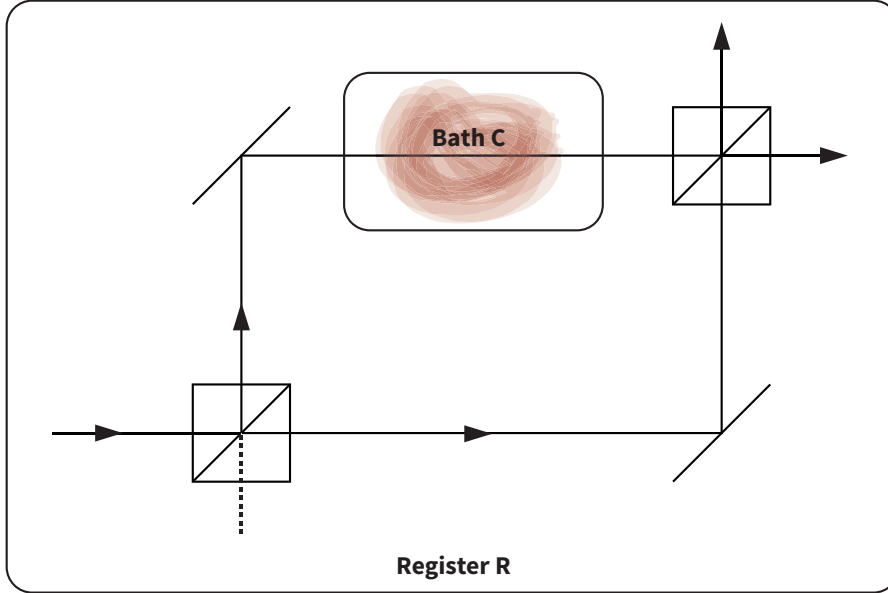
Figure 7: Interferometric implementation of a thermal hourglass where bath C is used as a cursor

The hourglass provides an adequate setting for an analysis of such a possible thermal time measuring device. An interferometric approach is depicted in figure 7. Here a well-controlled thermal bath, e.g. consisting of M non-interacting two-level systems, is used as a cursor system C, s.t. the state is given by

$$\rho_C = \frac{e^{-\beta H}}{Z} \tag{53}$$

where $\beta = \frac{1}{k_B T}$ is the inverse temperature and $Z = \text{Tr}\left(e^{-\beta H}\right)$ denotes the partition function. The register system R is similarly given by N non-interacting two-level systems. Here $N \ll M$ is needed in order not to disturb the bath too much. A suitable interaction can be given by the Hamiltonian presented in [75]

$$H_{int} = \alpha \sum_{k=1}^{M} \sum_{j=1}^{N} |1\rangle\langle 1|_{C_k} \otimes |1\rangle\langle 1|_{R_j} \tag{54}$$

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where α is the square root of the expected number of photons n in a coherent pulse $|\alpha\rangle$ defined as $|\alpha|^2 = n$. Through the mutual interaction of the bath (cursor) and the register, the state of register in the upper interferometer arm gains a phase $\phi = \alpha m t$. Here m is the number of systems in the excited state and is assumed to be known. It is a prominent result in quantum metrology that the ultimate limit for the precision of the measurement of the phase ϕ is given by the Heisenberg-limit [76, 77], i.e. $\sigma_\phi \propto \frac{1}{N}$. Thus knowing α and m precisely one can estimate t with the same precision.

Another interesting approach would be to take two thermal baths for the cursor system. A schematic sketch is given in figure 8. Instead of trying to distinguish the two different temperatures of the baths as chosen in [74], the distinguishability of the states given by the interaction with either bath 1 or bath 2 is used to measure time. The distinguishability Δ in terms of the trace distance is given by

$$\Delta := \frac{1}{2} \left\| \rho_1(T_1, t) - \rho_2(T_2, t) \right\|_1 \tag{55}$$

where ρ_1 and ρ_2 are the states after the interaction with bath 1 or bath 2 respectively. Essentially this approach works as the Δ shows a peak [74] defining a time t unambiguously depending solely on the temperatures T_1 and T_2. The workings of this thermal hourglass can be understood as distinguishing between two completely positive maps $\mathcal{E}_1$ and $\mathcal{E}_2$.
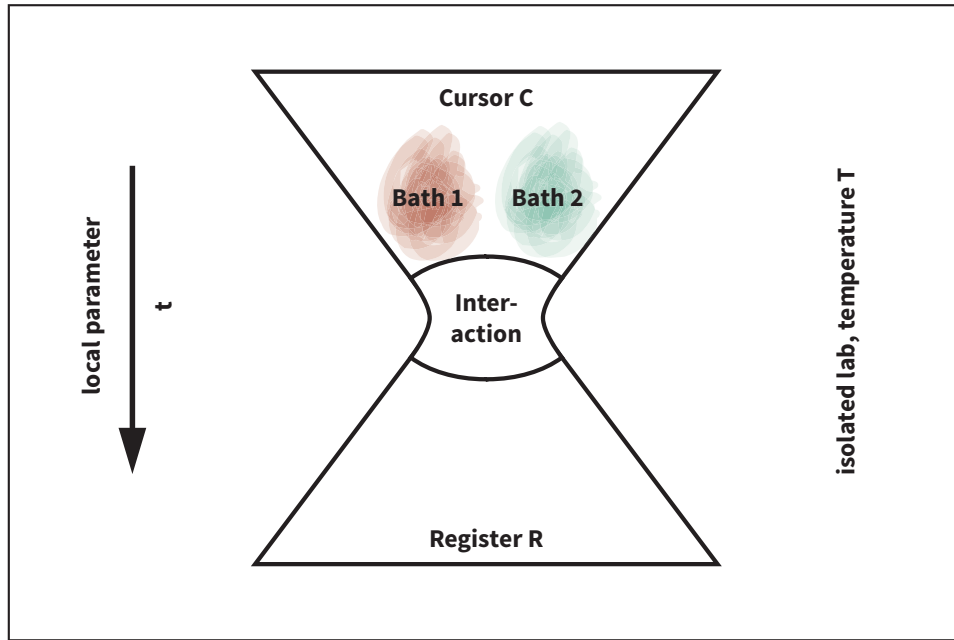
Figure 8: Thermal hourglass where the cursor system C consists of bath 1 and bath 2 in an isolated lab with temperature T

Yet another approach that will just be mentioned and not be discussed further, is to take both the cursor system C and the register R to be given by thermal baths. Two schematic illustrations are by figure 9 and figure 10. How well such an hourglass is able to measure time or what conclusions can be drawn from such an approach about the relation of time and temperature remains open for future research.
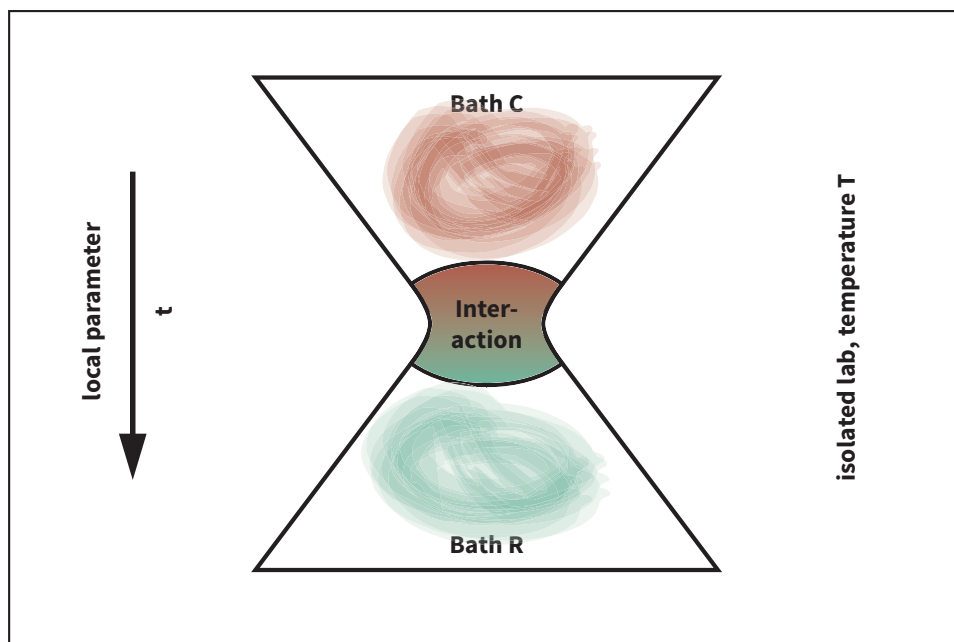
Figure 9: Thermal hourglass with cursor bath C and register bath R where $T_C \geq T_R$ in an isolated lab with temperature T

As a closing remark for this topic it is noted that thermodynamic interactions are far from being adequately well-controlled in experiment in order to provide time resolutions and precision/accuracies comparable to other forms of interaction. This is mirrored by the fact that the value of the Stefan-Boltzmann constant k_B is the one of least precisely measured fundamental constants. Only Newton's gravitational constant G surpasses k_B in relative uncertainty (based on CODATA set 2010 [9]).
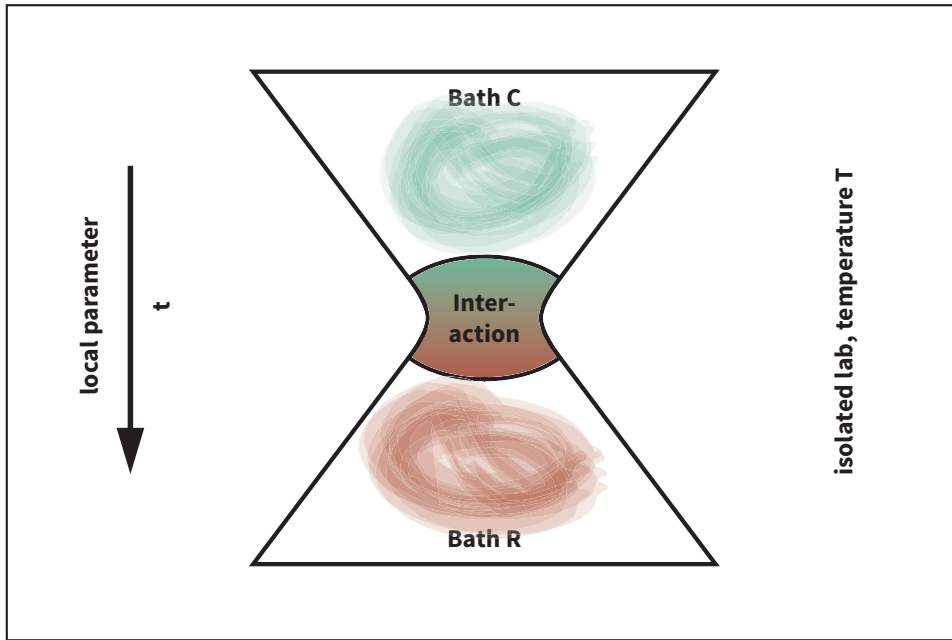
Figure 10: Thermal hourglass with cursor bath C and register bath R where $T_R \geq T_C$ in an isolated lab with temperature T

3.4 de Broglie hourglass

Since de Broglie wrote his Nobel prize winning Ph.D. thesis in 1924 we know that matter can be viewed as a wave [78]. His hypothesis was soon confirmed by the electron diffraction experiments of Davisson and Germer [79] yielding a shared Nobel prize for both of them. How does this relate to the hourglass? Well, already in his Ph.D. thesis de Broglie conjectured that a particle at rest with energy $E = m_0 c^2$ has an internal clock with a frequency

$$\nu_0 = \frac{m_0 c^2}{h}. \tag{56}$$

Hestenes linked this ideas to the Zitterbewegung (half of the former) suggested by Schrödinger [80] in series of papers pleading in favor of non-metaphorical interpretation of the former [81]. Whatsoever interpretation

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of this frequency one might think of, making use of it seems very appealing. Not only would it fundamentally link the units of mass and time (the SI unit kilogram is still defined with respect to the international prototype kilogram in Paris [82]) given a fixed value of $\hbar$, the access to such a high frequency would also enable time resolutions order of magnitudes higher then the one used for today's definition of a second [83] defined solely by the particle's mass.

Unfortunately present-day technology has no direct access to such high frequencies yet. It might even be that the de Broglie frequencies may be unobservable. A strong point against this pessimistic view was made Müller and his collaborators from the UC Berkeley's Department of Physics [84]. Their claim that they have gained access to the de Broglie frequency by means of an atom interferometer has resulted in a heated subsequent discussion (which certainly is fertile for our collective scientific enterprise). Their antagonists including Bordé and Cohen-Tannoudji assert that the interferometer under consideration is actually no atomic clock [85, 86]. As an atomic clock oscillating at de Broglie frequency would need two states in the two arms of the interferometer, that differ at least by mc^2. Most of their discussion evolves around whether the mentioned atomic interferometer, denoted compton clock (the name certainly bears an experimentalist bias) by Müller and his collaborators, is able to test the gravitational redshift or not. As this is out of the scope of this thesis, the interested reader is referred to the following references for the discussion [85, 86, 87, 88, 89] and a proposed resolution of the controversy [90].
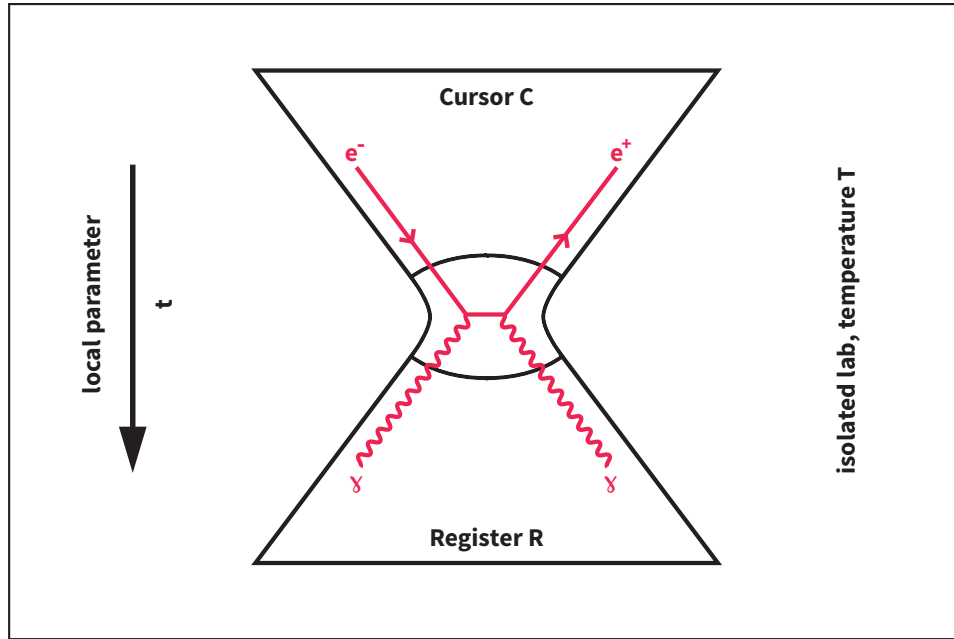
Figure 11: Matter-antimatter clock - de Broglie hourglass in an isolated lab
with temperature T

In this thesis an optimistic standpoint is taken conjecturing that a de Broglie
hourglass is possible, i.e. a time measuring device that is referenced to the
de Broglie frequency of a particle. Such an hourglass would need a suit-
able interaction allowing the register to record the evolution of the cursor
system. One could think of the scenario where it is technically possible to
control electron-positron annihilation, in such a case the produced photon
pair with frequencies referenced to the particle rest mass (~ 0.5 MeV) could
then be taken as a register system. This in principle even works for heavier
particles but the achievability of this approach depends highly on our ability

to directly or indirectly measure high frequencies ($\sim 10^8$ THz). Figure 11 sketches a possible matter-antimatter clock/de Broglie hourglass.

Note different other experimental attempts to access or simulate the de Broglie frequency/Zitterbewegung in [91, 92, 93].

3.5 Relativistic considerations

The intricate role that time plays in unifying quantum theory with other well-established fields in physics such as Relativity Theory and Thermodamics was already briefly pointed out in the previous sections. Particularly its notion seems to be at the core of the problem in formulating a quantum theory of gravity. This has been pointed out repeatedly [94, 95, 96, 97]. Nonetheless this section covers possible relativistic effects on the quantum hourglass.

So far the discussion of relativistic effects on the quantum hourglass was omitted, persisting upon the definition of the hourglass as a non-relativistic model. This assumption is quite unsound and was only made to simplify the analysis of the hourglass given so far. If spacetime is assumed to be classical one can work within Quantum Field Theory, allowing for reasoning about gravitational and motional effects on the hourglass, describing matter and light by quantized fields. In [98] the authors study a cursor system[2] given by a single localized quantum field. This quantum field undergoes a period of non-uniform acceleration. Then the attainable precision is calculated in terms of the Quantum Fisher Information using recently developed techniques of relativistic quantum information [99, 100]. Their results show that while in the absence of motion a squeezed vacuum state is the best

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[2]The cursor C needs to be accessed in some way in order to gain information, thus establishing the necessity of a register system R.

choice, coherent modes are more robust against the degradation of precision due to the non-uniform acceleration. Furthermore they propose testing the results obtained with superconducting resonators by hyper-fast tuning of the boundary conditions. A technique already proposed in [101] and [102] for earth-based experiments simulating relativistic effects in superconducting circuits.

In a regime sufficient for all today's practical purposes Relativistic Quantum Information seems to provide good approximations. Still it is not quite satisfying having to make the compromise of assuming a classical spacetime without knowing the mechanisms of its emergence. A possible mechanism that achieves the emergence of classicality without assuming a breakdown of Quantum Theory was proposed by Vienna based physicists Pikovski, Zych, Costa and Brukner in [103]. For this purpose they derive an effective Schrödinger equation that incorporates general relativistic corrections due to time dilation. The proposed decoherence due to time dilation arises even when there is no (known) coupling to an external environment. That means that one cannot shield the hourglass against this kind of decoherence without taking it to a spacetime region with no time dilation. For microscopic hourglasses this may not be of big importance but for hourglasses that use composite systems of gram scale this means that no large vertical superposition can be sustained even on a microsecond scale. Note also that in this mechanism the decoherence time depends on the temperature therefore it can be considered being a genuinely relativistic, thermodynamic and quantum effect [103].

Another relativistic effect proposed by the same group of physicist can entail serious consequences for an hourglass that is implemented as an interferometer (as e.g. the thermal hourglass in section 3.3). In [104] they show that using interferometer in a gravitational field the visibility is reduced if the particle has a degree of freedom that can be considered a 'clock' (in the context

of this thesis corresponding to the cursor system C, see footnote 3). They phrase the interferometric visibility $\mathcal{V}$ solely in terms of the time dilation between the trajectories τ and $t_\perp$,

$$\mathcal{V} = \left| \cos \left(\frac{\tau}{t_\perp} \frac{\pi}{2} \right) \right|. \tag{57}$$

Thus if the time dilation τ is equal to $t_\perp$ which means that the proper time difference is physically accessible, the interference fringes vanish. In the case that one does not consider a degree of freedom that can be seen as a 'clock'/cursor, the visibility is always maximal, as no which-path information is available. Furthermore they propose that this effect can be used in order to test whether proper time is a new quantum degree of freedom [104].

Note that the limitations on the measurement of spacetime distances have already been explored in the late 50's by Salecker and Wigner [105, 107]. In conclusion it is indicated that above relativistic considerations about the hourglass are surely not exhaustive. Which for now, in a time where no generally recognized unification of Relativity and Quantum Theory has been brought to the table yet, is simply not possible. Still an important point was made, namely, that a careful analysis incorporating relativistic effects is indeed necessary even in situations where one naïvely might think assuming a non-relativistic framework is sound.

4 Synchronization and Causality

> This idea that there is
> generality in the specific is of
> far-reaching importance.
>
> Douglas R. Hofstadter [108]

When one takes scenarios into account where there is more than one hourglass, the analysis becomes more intricated especially if the hourglasses reside in seperated labs. Quantum Information contains powerful tools and concepts that are well suited to analyze such scenarios. One example would be to formulate tasks as games allowing different strategies to be compared in terms of their winning probability. But before we can start the digression into the topics of synchronization and causality, it has to be made clear that these questions very much depend upon what resources one is allowed use. Especially in the case of synchronization the assumptions chosen should be made evident in order to enable one to put results and ideas into the right context.

First of all one has to mention that as soon as one takes communication into account, the assumption of the isolated lab in the hourglass model is relaxed in the following sense. For the events of receiving or sending a system from/to other agents the laboratory needs to be opened, in between such events however it remains isolated from the rest of the universe. Moreover the only assumption made is the local validity of Quantum Theory. In principal no global properties of spacetime or any predefined global ordering in terms of causality needs to be assumed [3]. Indications that such assumptions are sensical, in order to gain the ability to check fundamental properties of the world we live in, were given by results in the recent past showing that

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[3]This is in accordance with the framework proposed by Oreshkov, Costa and Brukner [111] proposed in order to discuss situations with no predefined causal order.

(within reasonable assumptions) the structure of the probabilistic theory underlying Quantum Theory cannot be modified [109, 110][4].

One of the most restrictive cases one can think of to analyze the synchronization of hourglasses is to allow for no communication between them, testing the fundamental limitations of their ability to remain synchronized in a separable fashion.
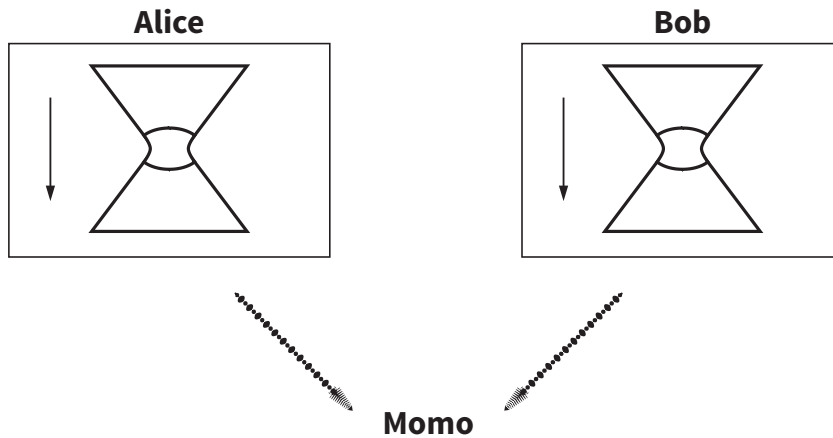

Figure 12: Schematic representation of a basic scenario for a synchronization game with two parties

The simplest case would be to take only two parties into account. In the schematic illustration drawn in figure 12 Momo acts as referee, deciding whether Alice's and Bob's hourglasses are synchronized. In such a setting Ranković, Liang and Renner show in [35] that one can place a bound on the number of synchronized 'ticks' produced. Assuming the existence of a global background parameter t they achieve a bound that is stated solely in terms

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[4]One possible modification would be to extend or replace Quantum Theory with a more general probabilistic theory. In [109] this would result in either the need for dropping the assumption of free choice or the validity of relativistic spacetime. [110] shows that such a modification is not possible without changing the dimensionality of space.

of the dimension of the cursor system d_C and a quantity characterizing the quality of synchronization. Moreover they propose a specific synchronization game [35] based on this very scenario in order to gain the ability to construct a global ordering scale that could replace our present ambiguous notion of time. This is a very interesting line of research. The next logical step would be to go beyond the two-party scenario. To look at multipartite scenarios (depicted in figure 13) seems to be necessary in order to make conclusive statements. Indications in this direction can be found in the next paragraph.
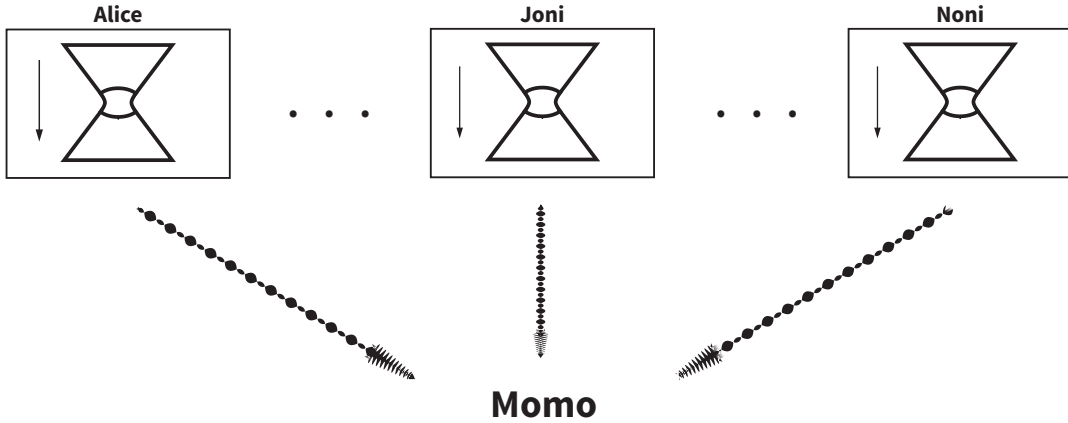

Figure 13: Schematic representation of a basic scenario for a synchronization game with $n > 2$ players

We will see in the following short story conveyed about the recent history of Causality Theory[5] that the establishment of two-party results does not necessarily allow for statements in n-player scenarios (with $n > 2$). In 2011 the framework proposed by Oreshkov, Costa and Brukner (OCB) [111] allowed them to arrive at astonishing results, namely, they found correlations that do not admit a definite causal order, thus violating a causal inequality.

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[5]For the interested reader that is not satisfied the following references are recommended [150, 151, 152, 153, 154, 155].

Moreover they showed that in this two-party scenario causal structure automatically arises in the classical limit. The topic was taken up by Lugano based scientists Baumeler and Wolf complementing the OCB framework with two tripartite non-causal games early this year [112]. Not long after that Baumeler, Feix and Wolf surprisingly put the multipartite question to rest showing that in the tripartite case the statement that predefined causal order always emerges in the classical limit does not hold true [113]. This adds once more an interesting peculiarity to the 'not predefined yet correlated'[113] connection of systems that initially disturbed Einstein, Podolski and Rosen so much that they decided to consider Quantum Theory as incomplete [23]. As there exists an seemingly infinite amount of literature on the very topic of such correlations here the interested reader is referred to a drastically limited personal selection of references [23, 114, 115][6].

As a last part of this section a recent paper in which the authors propose a quantum network of 'clocks' will be discussed [116][7]. Analyzing the scheme proposed in [116] in the new context of the hourglass one finds that the scheme should rather be seen as establishing one clock/hourglass with a register/cursor that manifests in different space-liked separated locations but that is still correlated due to the use of entanglement. Thus it can be seen as an example were one uses the results presented in section 2.2. This means using M coupled channels to transmit M times more bits in the same time $t_\perp$ given the same power P as it would be possible with a single qubit channel. The drawback of the scheme they propose is that an exponential scaling use of entanglement is necessary in order to attain and sustain a Heisenberg scaling for this world clock/hourglass [116].

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[6]The author of this thesis is pretty sure that iterating a process where one takes into account the bibliography of the cited publications and articles that refer to the cited publications, the set of reasonable publications on this topic that cannot be found by this 'algorithm' is of measure zero.

[7]Note that so far no specifications about the size of the laboratory have been made such that one may as well take it to be given by the rest of the universe.

5 The Future

The future is unwritten.

The Clash [106]

The hourglass was defined so general it basically contains nearly any system. This was done on purpose to achieve the ability to analyze different types of clocks in different physical and information-theoretical scenarios within the same framework. It is also very useful as any system can serve as a clock [117]. Some serve their purpose well, some don't, depending on the set of requirements. For example me playing the drums would not be a well suited system for a globally synchronized time needed to operate the United States government's global positioning system. Thus one of the most urging questions "What is a good clock and how can we characterize it?" still remains open. An interesting approach to tackle this problem would be to construct a resource theory (see [118] for an interesting paper that appeared last week and today's [119]) for hourglasses. Maybe this is not yet attainable in general but one could start to do so for a given set of scenarios. Interesting work in this direction has already been done by Janzing and Beth [120]. Such that one could also start the task by revisiting the extensive work done by both authors on timing information in the light of the quantum hourglass.

The hourglass was meant to serve as a way to model/implement an autonomously working clock that may be part of a microscopic machine in need of timing information. Having in mind that the hourglass model is computationally universal one particular interesting direction for further research are certainly quantum celular automata [121, 122, 123]. Except for wondering what the halting problem [124] means for an hourglass, note that in such microscopic scenario the assumption of the isolated lab seems a little to idealistic such that one probably also has to consider cases where the

overall state is not pure anymore, i.e.

$$\rho_{CRL} \in \mathcal{P}(\mathcal{H}_{CRL}). \tag{58}$$

However I think that one should not limit the potential of the hourglass model by just taking into account microscopic considerations. This statement stems from the observation that situations on very small scales and very large scales can inherit duality [125]. Thus I think that the hourglass may also be of importance for quantum cosmology [126, 127]. In my personal opinion I see the universe itself as the largest hourglass we can imagine/observe.

Another promising approach, I could imagine, is to employ a very powerful technique called compressed quantum sensing [128, 129, 130, 131]. This follows the following intuition: The optimal interaction for an hourglass is to have no interaction. In this way the cursor system C could evolve freely without being disturbed. The problem is that if we have no interaction we also have no information about the cursor. Here compressed sensing comes into play providing a way to make use of the sparse data provided through an minuscule interaction. These techniques could probably also come in handy if one thinks of using vacuum fluctuations as a cursor system [132]. A schematic illustration of such a possible vacuum hourglass is given in figure 14.
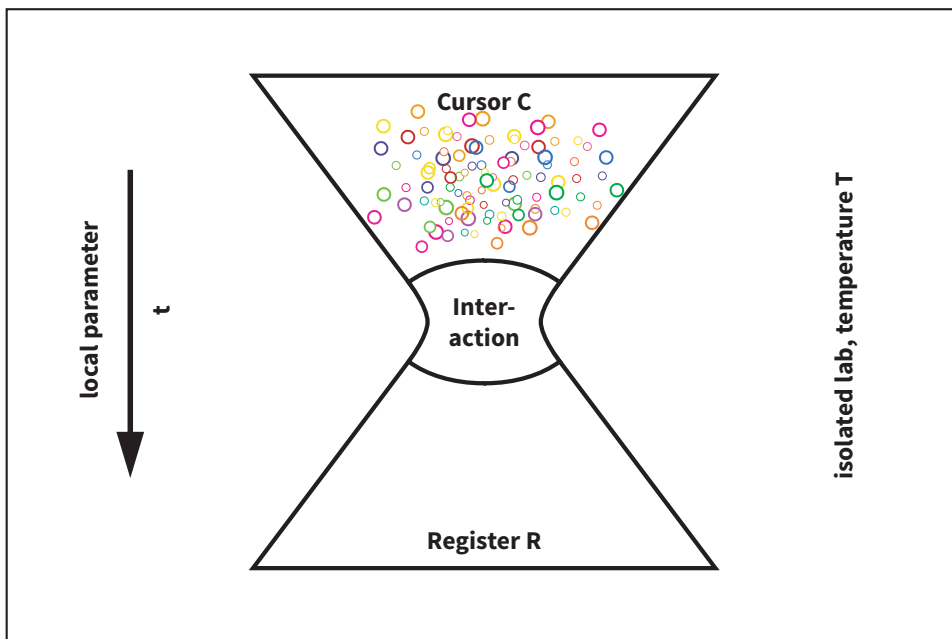
Figure 14: Quantum hourglass where vacuum fluctuations are used as a cursor system C

So far adiabaticity was only shortly addressed in the beginning in this thesis. Here we want to catch up a little on this topic as it brings along interesting questions. One might for example wonder how the relation between Hamiltonian Quantum Computing and Adiabatic Quantum Computing relates to the quantum hourglass. Moreover recent results obtained on super-adiabatic quantum engines might prove crucial for the implementation of the quantum hourglass [133]. Further research in this direction should certainly be conducted in the future.

In section 3 the possibility of an hourglass equipped with a memory system M was stated. I imagine such an additional operational part could be helpful in various situations. To give an example, consider the case where the memory M is correlated with the cursor C. From a quantum information

point of view this allows for the establishment of an entanglement assisted channel. Another example is to consider correlations between the memory M and the register R. In such a setting one could guarantee that no information is passed on from the register R to the cursor C through e.g. monogamy of entanglement. Maybe such hourglasses (depicted in figure15) are not implementable yet but they can still serve as theoretical playground for further research.
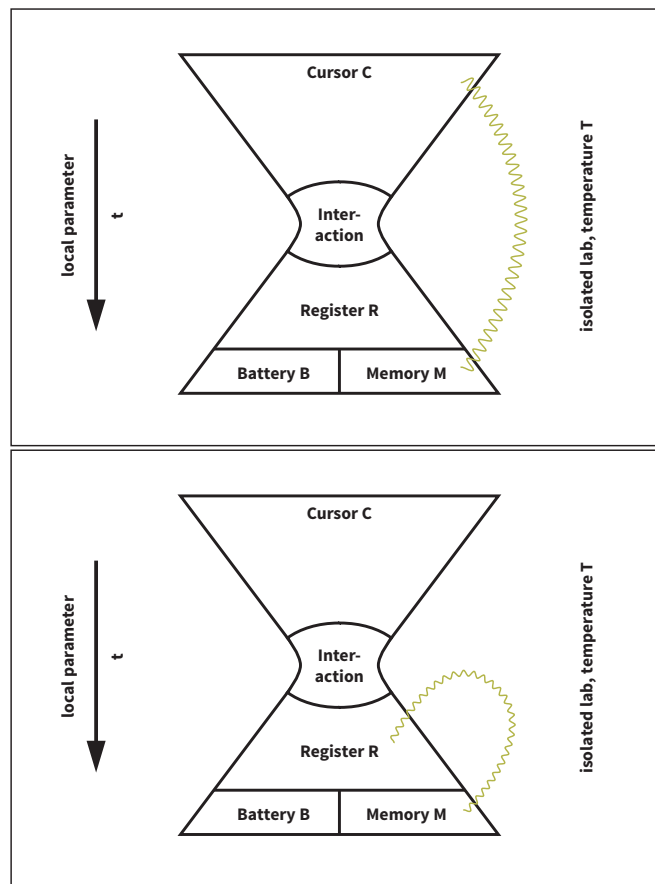

Figure 15: Quantum hourglass where correlation between the memory system M and the cursor system C (top)/the register system R (bottom) are available

The future is probably also the right time to think about how to use our correlations, i.e. time. Examples for such can be found in the literature exhaustively, see e.g. [134, 135, 136, 137, 138, 139, 140, 141].
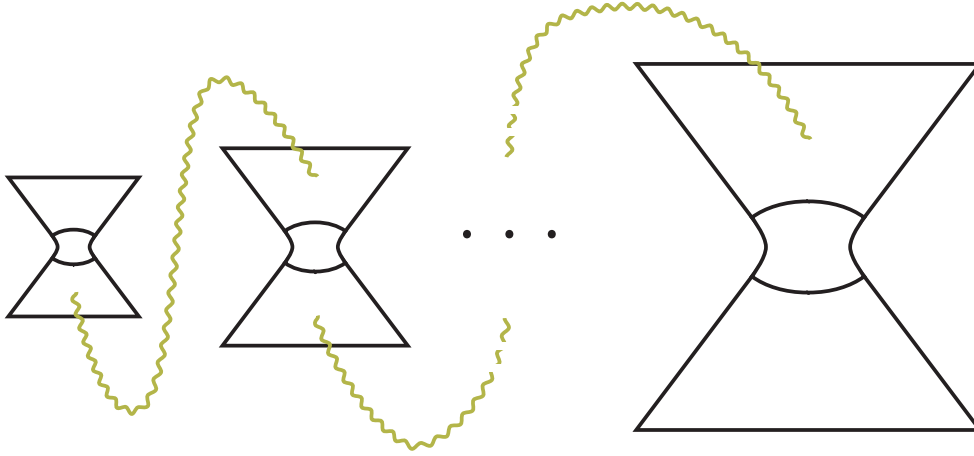

Figure 16: Quantum chronometer - chain of hourglasses connected via periodic entanglement of scale (see text)

Finally I want to touch on a highly speculative idea that could boost the time resolution of an hourglass somehow similar to the workings of the gears in a bicycle or mechanical watch. This idea of a quantum chronometer is depicted in figure 16. A necessary condition for such a device would be that one is provided with an periodic entanglement of scale. By periodic entanglement of scale I mean that for example the register of hourglass 1 is coupled to the cursor of hourglass 2 in such a way that it triggers cursor 2 to tick after itself having ticked 5 times. The previous example would result in the quantum analog to a 5:1 gear. Each hourglass should be ticking at the maximum rate (given by the Margolus-Levitin or a generalized bound) such that in the end reading of the last register in this chain would enable one to measure time with a higher time resolution (determined by the different gear ratios) than actually possible in the accessed hourglass standing

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by itself. It is an entanglement of scale as the two degrees of freedom entangled live on different (energy) scales. So far this is pure speculation, but it would be highly desirable to either be able to engineer such a mechanism or to find it appearing naturally in some part of our microscopic environment.

Last but not least I want to mention that in the future definitely more attention should be paid to the Zych-Zielinski-Kosinski bound (generalized Margolus-Levitin bound)

$$t_\perp \geq \frac{\pi \hbar}{2^{\frac{1}{\alpha}} \langle \psi | E^\alpha | \psi \rangle^{\frac{1}{\alpha}}}, \tag{59}$$

than I was able to provide in this thesis.

6   Conclusion

> ... white chocolate soup with mint ...
>
> ————————————————
>
> Julien Degorre [142]

First an historic, mathematical and conceptual context was set in order to establish a general operational model for quantum clocks called quantum hourglass or short hourglass. Its ability to distinguish 'coarser and finer' time sequences in different physical and information-theoretic scenarios was discussed. Therefore different types of hourglasses were constructed. It was shown that for cases where the interaction between the two parts of the quantum hourglass can be modeled by a qubit channel the time resolution of the quantum hourglass is limited by its power consumption. Furthermore it was demonstrated that the hourglass is computationally universal as it allows for the implementation of an universal set of gates. Thus establishing a connection between computation and time. A section was dedicated to a short digresion into the realms of synchronization and causality, scenarios where such a model as the hourglass may prove very useful. In the section named 'Future' open problems were posed, some intuitions were considered and plain speculations were contemplated on.

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[143, **?**, 145, 146, 147, 149]

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